GENERATION PRICING FOR A SINGLE AREA POWER SYSTEM USING MODIFIED SECOND ORDER NEWTON-RAPHSON TECHNIQUES

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by
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CERTIFICATE

This is to certify that the work contained in this thesis titled **Generation pricing for a single area power system using modified second order Newton-Raphson techniques** submitted by K Hari Chakrapani has been carried out under my supervision and that has not been submitted elsewhere for a degree.

Feb 2001

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ABSTRACT

The Newton-Raphson method is a powerful tool to solve the non-linear equations or simultaneous equations. It is popular due to its fast convergence properties. The modifications suggested in literature over standard NR is to accommodate ill-conditioning and improve the speed of convergence. Large number of variations of NR suggested in literature are only for single non-linear equation. However, it has been observed that it is not possible to extend these methods to a set of non-linear equations in straightforward fashion. This thesis work presents various modifications of NR for solving set of non-linear equations. Sudden change in load in power system will cause change in frequency as well as real power demand. Cost of generation is made as a function of frequency and real power demand. Present thesis work integrates both these changes and provides a mathematical framework for calculation of generation pricing.

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CHAPTER 1

INTRODUCTION

1.1 GENERAL

Set of non-linear equations can be solved in different ways, there are different approaches in literature. In which it is very true that Newton-Raphson method is the most popular, robust and efficient method. It too has its drawbacks like divergence when Jacobian becomes singular and taking more number of iterations when initial guess is far away from the final solution.

There are several variations of Newton-Raphson methods available in literature [1,2]. These variations mainly handle two problems i.e. speed of convergence and total computations required and ill conditioning of Jacobian (first derivative terms). The Newton-Raphson methods are based on Taylor's series. The number of terms considered in the series generally improves the order of convergence, as well the approximation of function but adds to total computation required. Further, inclusion of higher derivatives contributes to non linearity of method. There are several approaches to convert nonlinear Newton-Raphson to linear Newton-Raphson methods and these methods can be applied to non-linear equations with single variables only.

However the extension of higher order Newton-Raphson method to obtain solution for a set of non-linear equations is not a straight forward procedure due to coupling among variables and non linearity of the Newton-Raphson method. Modification of higher order Newton-Raphson method are proposed [3] in the present thesis to handle set of non linear equations and they are applied to power system load flow problem.

Load flow solution is a solution of non linear equation of a network under steady state condition subjected to certain inequality constraints. Newton-Raphson method is very useful to solve this problem, First order Newton-Raphson method will fail to converge when the system is ill-conditioned one such condition is R/X greater than 1 and

when the Jacobian matrix becomes singular, Modified Newton-Raphson methods are used to handle this condition in the present thesis work.

Change in real power at any bus in a power system will increase the load on system generator which will directly affect the system frequency and tie line loadings. The generation price will change with these changes. To find out the frequency deviation and generation pricing Automatic Generation Control [4] is been used in the current work. The automatic generation control used in this work considers all the essential components of power system like Governor, Turbine, Generator and Controllers.

In real life load cannot be treated as constant there will be fluctuation which will cause the generator to adjust their generation time to time, so the generation price will not be constant and it can not be divided into discrete intervals. A mathematical framework [5] has considered in this work, which will take care of both the frequency deviation and real power change. The objective is to increase price when frequency falls and vice-versa and to increase price when load increases and vice-versa.

1.2 ORGANISATION OF THESIS WORK

First order Newton-Raphson method has been explained in chapter 2. Its merits and demerits have been discussed. Modified Newton-Raphsons methods have been discussed and mathematical formulation for all these proposed methods has been given in this chapter and results for some non-linear algebraic problems have been given at the end of this chapter.

Application of Newton-Raphson method to power system load flow problem has been explained in chapter 3 and the methods proposed in chapter 2 are applied to this load flow problem and these methods are tested on some test systems like 13-bus system, 11-bus system and 14-bus system.

Generation pricing by considering both frequency deviation and power change has been explained in chapter 4. Mathematical frame work [5] for this method is given in this chapter. It is tested on a 31-bus radial system [5,6] in this work.

CHAPTER 2

NEWTON-RAPHSON METHOD

2.1 FIRST ORDER NEWTON-RAPHSON METHOD:

The Newton-Raphson method [7] is widely used for solving nonlinear equations. It transforms the original nonlinear problem into a sequence of linear problems whose solutions approach the solution of the original problem. The method can be applied to one equation in one unknown or to a system of simultaneous equations with as many unknowns as equations.

2.1.1 ONE DIMENSIONAL CASE:

Let F (x) be a nonlinear equation. Any value of x that satisfies F (x)=0 is a root of F (x). To find a particular root, an initial guess for x in the vicinity of the root is needed. Let this initial guess be x_0. Thus

$$F(x^{\circ}) = -\Delta F^{\circ} \quad \ldots \quad (2.1)$$

Where ΔF_0 is the error since x_0 is not a root. The situation can be shown graphically as in Figure 2.1. A tangent is drawn at the point on the curve corresponding to x_0, and is projected until it intercepts the x-axis to determine a second estimate of the root. Again the derivative is evaluated, and a tangent line is formed to proceed to the third estimate of x. the line generated in this process is given by

$$y(x) = F(x^k) + F'(x^k)(x - x^k) \quad \ldots \quad (2.2)$$

Which, when y (x)=0, gives the recursion formula for iterative estimates of the root:

$$x^{k+1} = x^k - \frac{F(x^k)}{F'(x^k)} \quad \dots \quad (2.3)$$

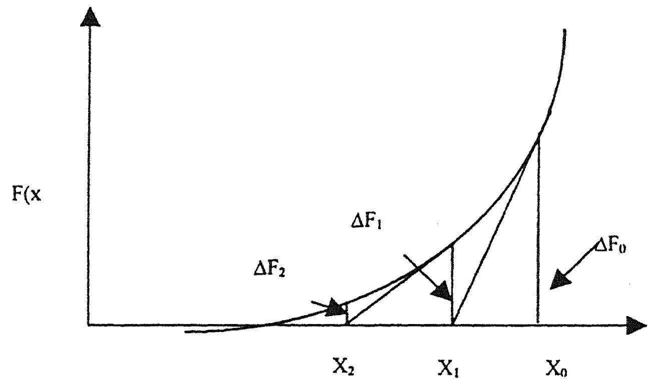


Fig 2.1 Illustrating the Newton-Raphson method

2.1.2 N-DIMENSIONAL CASE:

The single dimensional concept of the Newton-Raphson method can be extended to N dimensions. All that is needed is an N-dimensional analog of the first derivative. This is provided by the Jacobian (J) matrix. Each of the N rows of the Jacobian matrix is composed of the partial derivatives of one of the equations of the system with respect to each of the N variables.

FORMULATION OF N-DIMENSIONAL NEWTON-RAPHSON METHOD:

Let there are N non linear equations $F_1, F_2, \ldots\ldots, F_n$. Thus,

$$\begin{aligned} &F_1(x_1, x_2, x_3, \ldots\ldots, x_n) = 0 \\ &F_2(x_1, x_2, x_3, \ldots\ldots, x_n) = 0 \\ &F_3(x_1, x_2, x_3, \ldots\ldots, x_n) = 0 \quad \dots \quad (2.4) \\ &---- \\ &---- \\ &F_n(x_1, x_2, x_3, \ldots\ldots, x_n) = 0 \end{aligned}$$

Taylor Series expansion for F (x) will be

$$F(x)+F'(x)*\Delta x+\frac{1}{2}*F''(x)*\Delta x^2+.......=0 \quad ... \quad (2.5)$$

And for an N variable non linear equation, by neglecting third and higher order terms the Taylor Series expansion will be

$$F(x_1,...,x_n)+fx_1\Delta x_1+...+fx_n\Delta x_n=0 \quad ... \quad (2.6)$$

Where

$$fx_i=\frac{\partial F(x_1,...,x_n)}{\partial x_i} \quad i=1,2,...,n \quad ... \quad (2.7)$$

The Jacobian matrix for this N X N system is

$$\begin{bmatrix} \frac{\partial F_1}{\partial x_1} & \frac{\partial F_1}{\partial x_2} & .... & \frac{\partial F_1}{\partial x_n} \\ \frac{\partial F_2}{\partial x_1} & \frac{\partial F_2}{\partial x_2} & .... & \frac{\partial F_2}{\partial x_n} \\ ... & ... & .... & ... \\ ... & ... & .... & ... \\ \frac{\partial F_n}{\partial x_1} & \frac{\partial F_n}{\partial x_2} & .... & \frac{\partial F_n}{\partial x_n} \end{bmatrix} \quad ... \quad (2.8)$$

If the Jacobian matrix is numerically evaluated at some point $(x_1^0, x_2^0,....x_n^0)$, the following linear relationship is established for small displacements $(\Delta x_1, \Delta x_2,....,\Delta x_n)$:

$$\begin{bmatrix} \frac{\partial F_1^0}{\partial x_1} & \frac{\partial F_1^0}{\partial x_2} & .. & .. & \frac{\partial F_1^0}{\partial x_n} \\ \frac{\partial F_2^0}{\partial x_1} & \frac{\partial F_2^0}{\partial x_2} & .. & .. & \frac{\partial F_2^0}{\partial x_n} \\ .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. \\ \frac{\partial F_n^0}{\partial x_1} & \frac{\partial F_n^0}{\partial x_2} & .. & .. & \frac{\partial F_n^0}{\partial x_n} \end{bmatrix} \begin{bmatrix} \Delta x_1^1 \\ \Delta x_2^1 \\ .. \\ .. \\ \Delta x_n^1 \end{bmatrix} = \begin{bmatrix} \Delta F_1^0 \\ \Delta F_2^0 \\ .. \\ .. \\ \Delta F_n^0 \end{bmatrix} \quad ... \quad (2.9)$$

A recursive algorithm has been developed for computing the vector displacements $(\Delta x_1, \Delta x_2, ..., \Delta x_n)$. Each displacement is a solution to the related linear problem. With a good initial guess and other favorable conditions, the algorithm will converge to a solution of the nonlinear problem. Let $(x_1^0, x_2^0, ... x_n^0)$ be the initial guess. Then the errors are

$$
\begin{aligned}
\Delta F_1^0 &= -F_1[x_1^0, x_2^0, ..., x_n^0] \\
\Delta F_2^0 &= -F_2[x_1^0, x_2^0, ..., x_n^0] \quad ... \quad (2.10) \\
&...... \\
&...... \\
\Delta F_n^0 &= -F_n[x_1^0, x_2^0, ..., x_n^0]
\end{aligned}
$$

The jacobian matrix is then evaluated at the trial solution point $(x_1^0, x_2^0, ... x_n^0)$. Each element of the Jacobian matrix is computed from an algebraic formula for the appropriate partial derivative using $(x_1^0, x_2^0, ... x_n^0)$.

This system of non linear equations (2.4) is then solved directly for the first correction. The correction is then added to the initial guess to complete the first iteration:

$$
\begin{bmatrix} x_1^1 \\ x_2^1 \\ .. \\ .. \\ x_n^1 \end{bmatrix} = \begin{bmatrix} x_1^0 \\ x_2^0 \\ .. \\ .. \\ x_n^0 \end{bmatrix} + \begin{bmatrix} \Delta x_1^1 \\ \Delta x_2^1 \\ .. \\ .. \\ \Delta x_n^1 \end{bmatrix} \quad ... \quad (2.11)
$$

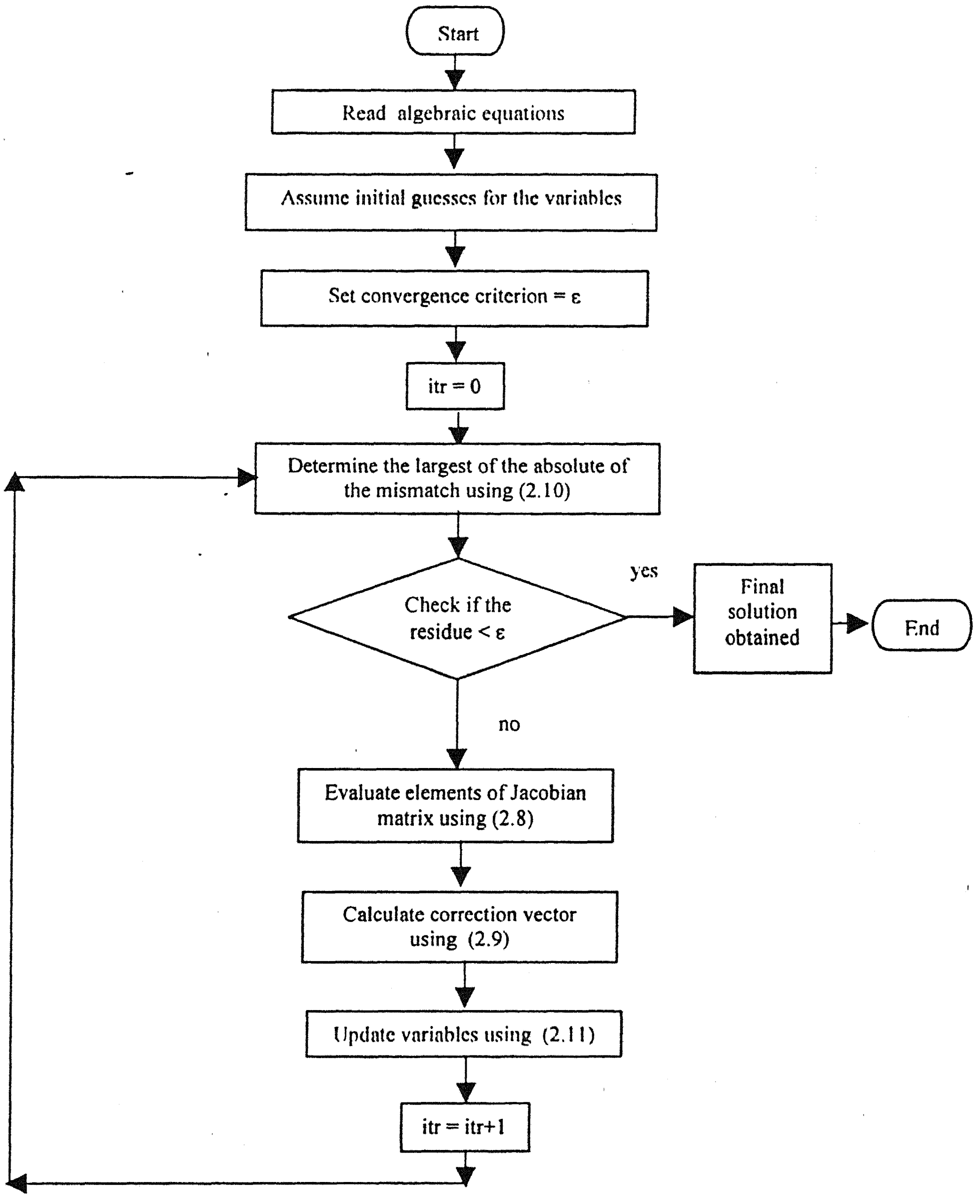
Equations (2.9) and (2.11) are rewritten using matrix symbols and a general superscript k for the iteration count:

$$
[J^{k-1}][\Delta x^k] = [\Delta F^{k-1}] \quad ... \quad (2.12)
$$

$$
x^k = x^{k-1} + \Delta x^k \quad ... \quad (2.13)
$$

The algorithm is repeated until ΔF^k satisfies some tolerance. In most solvable problems, it can be made practically zero. Tolerance value is 0.0001.

FLOW CHART FOR FIRST ORDER NEWTON-RAPHSON METHOD



2.2 MERITS AND DEMERITS OF FIRST ORDER NR METHOD

2.2.1 ADVANTAGES OF FIRST ORDER NEWTON-RAPHSON METHOD

- Newton-Raphson method is very powerful, using this any number of equations can be solved, which is not the case with Gauss-Siedal method.
- The convergence criterion used here is quadratic..
- Number of iterations are almost independent of number of equations.

2.2.2 DISADVANTAGES

- Solution we are going to get in First Order Newton-Raphson method depends upon the initial guess. If it is within a certain range, First Order will converge, but if it is far away from final solution it may fail to converge.
- If the Jacobian matrix formed becomes singular correction matrix cannot be calculated, as there will be no inverse for Jacobian.
- If the initial guess is far away from the final solution this First Order Method may take several iterations.

To overcome the above-mentioned disadvantages of First Order Newton-Raphson method various methods [1,2,3] have been implemented in the present thesis work, which will be described in the following sections. The methods [3] mainly concentrate on second order terms and the other authors have suggested some modifications to the First order Newton-Raphson method. In First Order Newton-Raphson method second order and higher order terms in Taylor series expansion are neglected. If second order terms are also taken into consideration they will introduce Hessian matrix along with Jacobian matrix.

2.3 SECOND ORDER NEWTON RAPHSON METHODS

2.3.1 N DIMENSIONAL SECOND ORDER NR METHOD

Let there are N non linear equations F_1, F_2, ..., F_n. Thus,

$$
\begin{aligned}
&F_1(x_1, x_2, x_3, ..., x_n) = 0 \\
&F_2(x_1, x_2, x_3, ..., x_n) = 0 \\
&F_3(x_1, x_2, x_3, ..., x_n) = 0 \quad \ldots \quad (2.14) \\
&---- \\
&---- \\
&F_n(x_1, x_2, x_3, ..., x_n) = 0
\end{aligned}
$$

Taylor Series expansion for F (x) will be

$$F(x) + F'(x) * \Delta x + \frac{1}{2} * F''(x) * \Delta x^2 + ... = 0 \quad \ldots \quad (2.15)$$

And for an N variable non linear equation, by neglecting third and higher order terms the Taylor Series expansion will be

$$F(x_1, ..., x_n) + fx_1 \Delta x_1 + ... + fx_n \Delta x_n + fx_1 x_1 \Delta x_1{}^2 + ... + fx_1 x_n \Delta x_1 \Delta x_n + ... + fx_n x_n \Delta x_n{}^2 = 0 \quad \ldots \quad (2.1$$

Where

$$fx_i = \frac{\partial F(x_1, ..., x_n)}{\partial x_i} \qquad i = 1,2,...,n \quad \ldots \quad (2.16)$$

$$fx_i x_j = \frac{\partial^2 F(x_1, ..., x_n)}{\partial x_j \partial x_i} \qquad i, j = 1,2,...,n \quad \ldots \quad (2.17)$$

$$fx_i x_i = \frac{\partial^2 F(x_1, ..., x_n)}{\partial^2 x_i} \qquad i = 1,2,...,n \quad \ldots \quad (2.18)$$

The Jacobian matrix for this N X N system will be

$$\begin{bmatrix} \frac{\partial F_1}{\partial x_1} & \frac{\partial F_1}{\partial x_2} & \cdots\cdots & \frac{\partial F_1}{\partial x_n} \\ \frac{\partial F_2}{\partial x_1} & \frac{\partial F_2}{\partial x_2} & \cdots\cdots & \frac{\partial F_2}{\partial x_n} \\ \cdots & \cdots & \cdots\cdots & \cdots \\ \cdots & \cdots & \cdots\cdots & \cdots \\ \frac{\partial F_n}{\partial x_1} & \frac{\partial F_n}{\partial x_2} & \cdots\cdots & \frac{\partial F_n}{\partial x_n} \end{bmatrix} \quad \ldots \quad (2.19)$$

The Hessian matrix for this N X N system will be

$$\begin{bmatrix} \frac{\partial^2 F_1}{\partial x_1^2} & \frac{\partial^2 F_1}{\partial x_2 \partial x_1} & \cdots & \frac{\partial^2 F_1}{\partial x_n \partial x_1} & \frac{\partial^2 F_1}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 F_1}{\partial x_q \partial x_p} & \cdots & \frac{\partial^2 F_1}{\partial x_n^2} \\ \frac{\partial^2 F_2}{\partial x_1^2} & \frac{\partial^2 F_2}{\partial x_2 \partial x_1} & \cdots & \frac{\partial^2 F_2}{\partial x_n \partial x_1} & \frac{\partial^2 F_2}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 F_2}{\partial x_q \partial x_p} & \cdots & \frac{\partial^2 F_2}{\partial x_n^2} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{\partial^2 F_n}{\partial x_1^2} & \frac{\partial^2 F_n}{\partial x_2 \partial x_1} & \cdots & \frac{\partial^2 F_n}{\partial x_n \partial x_1} & \frac{\partial^2 F_n}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 F_n}{\partial x_q \partial x_p} & \cdots & \frac{\partial^2 F_n}{\partial x_n^2} \end{bmatrix} \quad \ldots \quad (2.20)$$

The size of Hessian matrix will be n X n^2 by considering all the combinations , the size can be reduced to n X n*(n+1)/2 by making some changes in correction matrix as

$$\frac{\partial^2 F_i}{\partial x_q \partial x_p} = \frac{\partial^2 F_i}{\partial x_p \partial x_q}.$$

There will be two correction matrices for second order Newton-Raphson method. First correction matrix will be

$$\begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \\ \cdots \\ \cdots \\ \Delta x_n \end{bmatrix} \quad \ldots \quad (2.21)$$

Second correction matrix is

$$\begin{bmatrix} \Delta x_1.\Delta x_1 \\ 2\Delta x_2.\Delta x_1 \\ \dots \\ 2\Delta x_n.\Delta x_1 \\ \dots \\ \dots \\ 2\Delta x_q.\Delta x_p \\ \dots \\ \Delta x_n.\Delta x_n \end{bmatrix} \quad \dots \quad (2.22)$$

This correction matrix can be split into product of two matrices. One of these matrices will be first correction matrix. So the second correction matrix can be shown as W*first correction matrix. The matrix W will be

$$\begin{bmatrix} \Delta x_1 & 0 & 0 & .. & .. & .. & 0 \\ \Delta x_2 & \Delta x_1 & 0 & .. & .. & .. & 0 \\ .. & .. & .. & .. & .. & .. & .. \\ \Delta x_n & 0 & 0 & .. & .. & .. & \Delta x_1 \\ 0 & \Delta x_2 & 0 & 0 & .. & .. & 0 \\ 0 & \Delta x_3 & \Delta x_2 & 0 & .. & .. & 0 \\ .. & .. & .. & .. & .. & .. & .. \\ 0 & \Delta x_n & 0 & 0 & .. & .. & \Delta x_2 \\ .. & .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. & .. \\ 0 & 0 & 0 & .. & .. & .. & \Delta x_n \end{bmatrix} \quad \dots \quad (2.23)$$

The jacobian matrix, Hessian matrix and W matrix are evaluated at the initial guess $\left(x_1^0, x_2^0, \dots x_n^0\right)$. Mismatch vector can be calculated as

$$\begin{aligned}
\Delta F_1^0 &= -F_1\left[x_1^0, x_2^0, \ldots\ldots, x_n^0\right] \\
\Delta F_2^0 &= -F_2\left[x_1^0, x_2^0, \ldots\ldots, x_n^0\right] \\
&\ldots\ldots \\
&\ldots\ldots \\
\Delta F_n^0 &= -F_n\left[x_1^0, x_2^0, \ldots\ldots, x_n^0\right]
\end{aligned} \quad \ldots \quad (2.24)$$

Elements in W matrix can be calculated in different ways. For the first iteration they will be zero and hence for the first iteration the method will be just like first order Newton-Raphson method. The corrections in the first iteration will be used as elements for W matrix in second iteration and so on.

Let the mismatch vector be ΔF

Jacobian matrix be J

Hessian matrix be H and

Correction matrix be Δx

The second order Newton-Raphson method can be formulated as

$$\Delta F = J * \Delta x + \frac{1}{2} * H * W * \Delta x \quad \ldots \quad (2.25)$$

From the above equation correction matrix can be calculated as

$$\Delta x = \left(J + \frac{1}{2} * H * W\right)^{-1} * \Delta F \quad \ldots \quad (2.26)$$

By adding this correction vector to the initial guess new values of X can be calculated. In general this can be calculated as

$$X^{new} = X^{old} + \Delta X \quad \ldots \quad (2.27)$$

With these new values of X mismatch vector will be calculated. If these values are less than the tolerance value which will be generally 0.0001 it implies that the solution is very close to the final solution so that the iteration process can be stopped, otherwise again Jacobian, Hessian and W matrix are calculated with the updated

variables and same calculations has to be done as explained above, this has to carried out till mismatch vector elements become less than tolerance value.

Different second order Newton-Raphson methods are discussed below for convenience two variable case will be discussed.

Notations that will be used in the following methods

f, g	:	Functions, which will be taken as input
f_x	:	$\frac{\partial f}{\partial x}$
f_y	:	$\frac{\partial f}{\partial y}$
g_x	:	$\frac{\partial g}{\partial x}$
g_y	:	$\frac{\partial g}{\partial y}$
f_{xx}	:	$\frac{\partial^2 f}{\partial^2 x}$
f_{yy}	:	$\frac{\partial^2 f}{\partial^2 y}$
f_{xy}	:	$\frac{\partial^2 f}{\partial y \partial x}$
g_{xx}	:	$\frac{\partial^2 g}{\partial^2 x}$
g_{yy}	:	$\frac{\partial^2 g}{\partial^2 y}$
g_{xy}	:	$\frac{\partial^2 g}{\partial y \partial x}$
Δx	:	correction in variable x
Δy	:	correction in variable y

2.3.2 PROPOSED SECOND ORDER NR METHODS

2.3.2.1 METHOD-1:

From the Taylor's series expansion the equations f, g can be formulated as [3]

$$\begin{pmatrix} -f^k \\ -g^k \end{pmatrix} = \left(J^k + \frac{1}{2} * H^k * W^{k-1} \right) \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} \quad \ldots \quad (2.28)$$

Where

$$J = \begin{pmatrix} f_x & f_y \\ g_x & g_y \end{pmatrix} \quad \ldots \quad (2.29)$$

$$H = \begin{pmatrix} f_{xx} & f_{yy} & f_{xy} \\ g_{xx} & g_{yy} & g_{xy} \end{pmatrix} \quad \ldots \quad (2.30)$$

$$W = \begin{pmatrix} \Delta x & 0 \\ 0 & \Delta y \\ \Delta x & \Delta y \end{pmatrix} \quad \ldots \quad (2.31)$$

The elements in W matrix are obtained from previous iteration, they are nothing but the elements of the correction matrix of the previous iteration. For the first iteration they are treated as 0. With these matrices Newton-Raphson method will be carried out as explained in section (2.3.1) till occurrence of convergence. Convergence criterion for this method is all the elements of the mismatch vector have to become less than the tolerance value, which is 0.0001. Process will be stopped when the above-mentioned condition reached. Results for this method are shown at the end of this chapter. It has shown improvements in terms of number of iterations and initial guess wise.

2.3.2.2 METHOD 2:

In this method instead of taking previous iteration correction matrix elements into W matrix, they have calculate using the present iteration correction matrix as given below

$$W = \begin{pmatrix} -\dfrac{f^k}{f_x^k} & 0 \\ 0 & -\dfrac{g^k}{g_y^k} \\ -\dfrac{f^k}{f_y^k} & -\dfrac{f^k}{g_x^k} \end{pmatrix} \quad \ldots \quad (2.32)$$

The equation (2.28) can be written as

$$\begin{pmatrix} -f^k \\ -g^k \end{pmatrix} = \begin{pmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} \quad \ldots \quad (2.33)$$

Where

$$\begin{aligned} u_{11} &= f_x^k - \frac{1}{2}\frac{f_{xx}^k f^k}{f_x^k} - \frac{f_{xy}^k f^k}{f_y^k} \\ u_{12} &= f_y^k - \frac{1}{2}\frac{f_{yy}^k g^k}{g_y^k} - \frac{f_{xy}^k g^k}{g_x^k} \\ u_{21} &= g_x^k - \frac{1}{2}\frac{g_{xx}^k f^k}{f_x^k} - \frac{g_{xy}^k f^k}{f_y^k} \quad \ldots \quad (2.34) \\ u_{22} &= g_y^k - \frac{1}{2}\frac{g_{yy}^k g^k}{g_y^k} - \frac{g_{xy}^k g^k}{g_x^k} \end{aligned}$$

With these matrices Newton-Raphson method [3] will be carried out as explained in section (2.3.1) till occurrence of convergence. Convergence criterion for this method is all the elements of the mismatch vector have to become less than the tolerance value, which is 0.0001. Process will be stopped when the above-mentioned condition reached. Results for this method are shown at the end of this chapter.This method fails to converge in cases where the terms in the jacobian become zero. Because for calculating U matrix elements these Jacobian elements are coming in denominator. So this method will not be useful when the elements of Jacobian become zero.

2.3.2.3 METHOD 3:

This method can be formulated [3] as

$$\begin{pmatrix} -f \\ -g \end{pmatrix} = (J)\begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} + \frac{1}{2}(H)\begin{pmatrix} \Delta x & 0 \\ \Delta y & \Delta x \\ 0 & \Delta y \end{pmatrix}\begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} \quad \ldots \quad (2.35)$$

W matrix elements are calculated as

$$\begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} = J^{-1} * \begin{pmatrix} -f \\ -g \end{pmatrix} \quad \ldots \quad (2.36)$$

In this method new correction matrix is calculated as in First order Newton-Raphson method from the previous iteration values and fed as input into the W matrix. With these matrices Newton-Raphson method will be carried out as explained in section (2.3.1) till the variables converge to their optimal values. Convergence criterion for this method is all the elements of the mismatch vector have to become less than the tolerance value, which is 0.0001. Process will be stopped when the above-mentioned condition reached. This method requires less number of iteration and is robust with respect to the change of initial guess. This is the best method out of all the methods described here. Results are shown at the end of this chapter.

2.3.2.4 METHOD 4:

In this method instead of taking full Jacobian and full Hessian matrices some fraction of them are considered [3]. Let α be the relaxation parameter for Jacobian and β will be the relaxation parameter for Hessian matrix. By changing these values performance will be checked. This is a hit and trail method. If the summation of Jacobian and Hessian is becoming singular, the situation can be avoided by considering their fractions. The problem can be formulated as

$$\begin{pmatrix} -f \\ -g \end{pmatrix} = \alpha\,(J)\begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} + \beta\,(H)(W)\begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} \quad \ldots \quad (2.37)$$

With this α, β relaxation parameters and other matrices new mismatch vector is calculated as explained in section (2.3.1) till the system convergence. The convergence criterion is all the entries in mismatch vector have to be less than the tolerance value. The tolerance value is 0.0001. This has given mixed results. Choice of α, β is crucial to the efficiency of this method. Results for this method are shown at the end of this chapter.

2.3.2.5 METHOD 5:

A drawback of Newton-Raphson method is the computation time it will take to calculate Jacobian and Hessian matrices [3]. By keeping them constant in different combinations CPU time can be reduced. The elements of W matrix will be the correction matrix elements of the previous iteration and for the first iteration they will be zero. Four different combinations can be obtained while keeping these Jacobian and Hessian matrices constant.

2.3.2.5.1 METHOD 5(a):

In this method Jacobian will be kept constant and Hessian will be calculated in every iteration [3] and the procedure explained in section (2.3.1) will be followed to achieve the solution. The convergence criterion requires all the entries in mismatch vector have to be less than the tolerance value. The tolerance value is 0.0001.

2.3.2.5.2 METHOD 5(b):

In this method Hessian will be kept constant and Jacobian will be calculated in every iteration [3] and the procedure explained in section (2.3.1) will be followed to achieve the solution. The convergence criterion requires all the entries in mismatch vector have to be less than the tolerance value. The tolerance value is 0.0001.

2.3.2.5.3 METHOD 5(c):

In this method both Jacobian and Hessian have kept constant and the procedure explained in section (2.3.1) is followed to achieve the solution [3]. The convergence criterion requires all the entries in mismatch vector have to be less than the tolerance value. The tolerance value is 0.0001.

2.3.2.5.4 METHOD 5(d):

This is nothing but method-3, because we have calculated both Jacobian and Hessian in every iteration [3] and the procedure explained in section (2.3.1) is followed to achieve the solution. The convergence criterion requires all the entries in mismatch vector have to be less than the tolerance value. The tolerance value is 0.0001.

2.3.2.6 METHOD 6:

The above-mentioned methods will converge even if the Jacobian becomes singular. The term containing Hessian matrix, which is added to it makes the sum of Jacobian and Hessian non-singular and hence solution can be achieved [3]. If the Jacobian is well defined and to improve the convergence rate some modification are made in the basic equation so the new Newton-Raphson problem can be formulated as

$$\left[\begin{pmatrix} -f^k \\ -g^k \end{pmatrix} + \frac{1}{2}(H)\begin{pmatrix} \Delta x^{k-1}\ \Delta x^{k-1} \\ \Delta y^{k-1}\ \Delta y^{k-1} \\ 2\ \Delta x^{k-1}\ \Delta x^{k-1} \end{pmatrix}\right] = \left(J^k\right)\begin{pmatrix} \Delta x^k \\ \Delta y^k \end{pmatrix} \quad \ldots \quad (2.38)$$

Procedure for this method is second correction vector elements will be calculated with those of first correction vector's previous iteration values and the left hand side vector will be calculated as

$$\begin{pmatrix} \Delta x^k \\ \Delta y^k \end{pmatrix} = \left(J^k\right)^{-1}\left[\begin{pmatrix} -f^k \\ -g^k \end{pmatrix} + \frac{1}{2}(H)\begin{pmatrix} \Delta x^{k-1}\ \Delta x^{k-1} \\ \Delta y^{k-1}\ \Delta y^{k-1} \\ 2\ \Delta x^{k-1}\ \Delta x^{k-1} \end{pmatrix}\right] \quad \ldots \quad (2.39)$$

This correction vector elements are added to the old solution to achieve the new solution and the procedure explained in section (2.3.1) is followed to achieve the solution. The convergence criterion requires all the entries in mismatch vector have to be less than the tolerance value. The tolerance value is 0.0001.

2.3.2.7 <u>METHOD 7:</u>

In this method [3] the Hessian matrix has been split into two matrices for convenience purpose. Let the two matrices be H ^{1} and L. The entries in these two matrices are

$$H^1 = \begin{pmatrix} f_{xx} & f_{yy} \\ g_{xx} & g_{yy} \end{pmatrix} \quad \ldots \quad (2.40)$$

$$L = \begin{pmatrix} f_{yx} & f_{xy} \\ g_{yx} & g_{xy} \end{pmatrix} \quad \ldots \quad (2.41)$$

The problem can be formulated as

$$\begin{pmatrix} -f \\ -g \end{pmatrix} = (J)\begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} + \frac{1}{2}\left[\left(H^1 * W^1\right)\begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} + \left(L * W^2\right)\begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}\right] \quad \ldots \quad (2.42)$$

$$\begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} = \left(J + \frac{1}{2}\left(H^1 * W^1 + L * W^2\right)\right)^{-1}\begin{pmatrix} -f \\ -g \end{pmatrix} \quad \ldots \quad (2.43)$$

where

$$W^1 = \begin{pmatrix} \Delta x & 0 \\ 0 & \Delta y \end{pmatrix}$$

$$W^2 = \begin{pmatrix} \Delta y & 0 \\ 0 & \Delta x \end{pmatrix}$$

With the initial guess values Jacobian, H^1 and L matrices will be calculated and correction matrix will be calculated as shown in the equation (2.43). From this step the procedure will be as explained in section (2.3.1). Iteration will be continued till the occurrence of convergence. The convergence criterion requires all the entries in mismatch vector have to be less than the tolerance value. The tolerance value is 0.0001.

2.3.3 MODIFIED FIRST ORDER NEWTON-RAPHSON METHODS

2.3.3.1 NEWTON RAPHSON METHOD WITH DECELERATED CONVERGENCE

In this method [1,2] the correction vector is modified by multiplying with a factor 'α' which is less than 1.0, so that for ill-conditioned system the oscillations around the solution point may be reduced. The iteration scheme can be summarized as follows

$$\begin{pmatrix} \Delta x^{k+1} \\ \Delta y^{k+1} \end{pmatrix} = \left(J^k + \frac{1}{2} H^k W^{k-1} \right)^{-1} \begin{pmatrix} -f^k \\ -g^k \end{pmatrix} \quad \ldots \quad (2.44)$$

$$\begin{pmatrix} x^{k+1} \\ y^{k+1} \end{pmatrix} = \begin{pmatrix} x^k \\ y^k \end{pmatrix} + \alpha \begin{pmatrix} \Delta x^{k+1} \\ \Delta y^{k+1} \end{pmatrix} \quad \ldots \quad (2.45)$$

where α is in the range $0 \le \alpha \le 1$

2.3.3.2 NEWTON-RAPHSON METHOD WITH ACCELERATED CONVERGENCE

2.3.3.2.1 TWO STEP ALGORITHM

This method [1,2] can be summarized mathematically as

$$\begin{pmatrix} x^{k1} \\ y^{k1} \end{pmatrix} = \begin{pmatrix} x^k \\ y^k \end{pmatrix} + \left(J^k + \frac{1}{2} H^k W^{k-1} \right)^{-1} \begin{pmatrix} -f^k \\ -g^k \end{pmatrix} \quad \ldots \quad (2.46)$$

$$\begin{pmatrix} \Delta x^{k1} \\ \Delta y^{k1} \end{pmatrix} = \left(J^{k1} + \frac{1}{2} H^{k1} W^k \right)^{-1} \begin{pmatrix} -f^{k1} \\ -g^{k1} \end{pmatrix} \quad \ldots \quad (2.47)$$

$$\begin{pmatrix} x^{k+1} \\ y^{k+1} \end{pmatrix} = \begin{pmatrix} x^{k1} \\ y^{k1} \end{pmatrix} + \left(2 - C \left\| \begin{matrix} \Delta x^{k1} \\ \Delta y^{k1} \end{matrix} \right\|^{\alpha} \right) \left\| \begin{matrix} \Delta x^{k1} \\ \Delta y^{k1} \end{matrix} \right\| \quad \ldots \quad (2.48)$$

Where α and C are positive constants, $\left\| \begin{matrix} \Delta x^{k1} \\ \Delta y^{k1} \end{matrix} \right\|$ is the norm of the vector $\begin{pmatrix} \Delta x^{k1} \\ \Delta y^{k1} \end{pmatrix}$ and $\begin{pmatrix} x^{k1} \\ y^{k1} \end{pmatrix}$ is the intermediate solution vector.

2.3.3.2.2 THREE STEP ALGORITHM

This method [1,2] is an extension of the two-step algorithm and can be mathematically expressed as

$$\begin{pmatrix} x^{k1} \\ y^{k1} \end{pmatrix} = \begin{pmatrix} x^{k} \\ y^{k} \end{pmatrix} + \left(J^{k} + \frac{1}{2} H^{k} W^{k-1} \right)^{-1} \begin{pmatrix} -f^{k} \\ -g^{k} \end{pmatrix} \quad \ldots \quad (2.49)$$

$$\begin{pmatrix} x^{k2} \\ y^{k2} \end{pmatrix} = \begin{pmatrix} x^{k1} \\ y^{k1} \end{pmatrix} + \left(J^{k1} + \frac{1}{2} H^{k1} W^{k} \right)^{-1} \begin{pmatrix} -f^{k1} \\ -g^{k1} \end{pmatrix} \quad \ldots \quad (2.50)$$

$$\begin{pmatrix} \Delta x^{k2} \\ \Delta y^{k2} \end{pmatrix} = \left(J^{k2} + \frac{1}{2} H^{k2} W^{k1} \right)^{-1} \begin{pmatrix} -f^{k2} \\ -g^{k2} \end{pmatrix} \quad \ldots \quad (2.51)$$

$$\begin{pmatrix} x^{k+1} \\ y^{k+1} \end{pmatrix} = \begin{pmatrix} x^{k2} \\ y^{k2} \end{pmatrix} + \left(2 - C \left\| \begin{matrix} \Delta x^{k2} \\ \Delta y^{k2} \end{matrix} \right\|^{\alpha} \right) \left\| \begin{matrix} \Delta x^{k2} \\ \Delta y^{k2} \end{matrix} \right\| \quad \ldots \quad (2.52)$$

Where α and C are positive constants, $\left\| \begin{matrix} \Delta x^{k2} \\ \Delta y^{k2} \end{matrix} \right\|$ is the norm of the vector $\begin{pmatrix} \Delta x^{k2} \\ \Delta y^{k2} \end{pmatrix}$ and $\begin{pmatrix} x^{k1} \\ y^{k1} \end{pmatrix}$ $\begin{pmatrix} x^{k2} \\ y^{k2} \end{pmatrix}$ are the intermediate solution vectors.

2.3.3.3 CONSTANT POSITIVE SHIFT ALGORITHM

A new modification is proposed based on diagonal dominance by incorporating a shift [1,2]. This effect is brought about by giving a constant positive shift to all the diagonal elements of the Jacobian matrix. The iteration scheme for the proposed constant positive shift algorithm can be formulated mathematically as follows

$$\begin{pmatrix} \Delta x_{k+1} \\ \Delta y_{k+1} \end{pmatrix} = \left(J^k + \frac{1}{2} H^k W^{k-1} \right)_{\mathrm{mod}}^{-1} F^k \quad \ldots \quad (2.53)$$

$$\left(J^k + \frac{1}{2} H^k W^{k-1} \right)_{\mathrm{mod}} = \left(J^k + \frac{1}{2} H^k W^{k-1} \right) + S\, I \quad \ldots \quad (2.54)$$

$$F^k = \begin{pmatrix} -f^k \\ -g^k \end{pmatrix} + S \begin{pmatrix} \Delta x_k \\ \Delta y_k \end{pmatrix} \quad \ldots \quad (2.55)$$

$$\begin{pmatrix} x_{k+1} \\ y_{k+1} \end{pmatrix} = \begin{pmatrix} x_k \\ y_k \end{pmatrix} + \begin{pmatrix} \Delta x_{k+1} \\ \Delta y_{k+1} \end{pmatrix} \quad \ldots \quad (2.56)$$

Where I is the unit matrix and S is a positive constant.

2.3.3.4 INCREASING POSITIVE SHIFT ALGORITHM

This method [1,2] is also based on the principle of diagonal dominance of the Jacobian matrix. In this algorithm the positive shift of the Jacobian matrix is small at the beginning of the iteration process and increases as the iteration proceeds. This algorithm is formulated as follows

$$\begin{pmatrix} \Delta x_{k+1} \\ \Delta y_{k+1} \end{pmatrix} = \left(J^k + \frac{1}{2} H^k W^{k-1} \right)_{\mathrm{mod}}^{-1} F^k \quad \ldots \quad (2.57)$$

$$\left(J^k + \frac{1}{2} H^k W^{k-1} \right)_{\mathrm{mod}} = \left(J^k + \frac{1}{2} H^k W^{k-1} \right) + S_k\, I \quad \ldots \quad (2.58)$$

$$F^k = \begin{pmatrix} -f^k \\ -g^k \end{pmatrix} + S_k \begin{pmatrix} \Delta x_k \\ \Delta y_k \end{pmatrix} \quad \ldots \quad (2.59)$$

$$S_{k+1} = S_k\, C \quad \ldots \quad (2.60)$$

$$\begin{pmatrix} x_{k+1} \\ y_{k+1} \end{pmatrix} = \begin{pmatrix} x_k \\ y_k \end{pmatrix} + \begin{pmatrix} \Delta x_{k+1} \\ \Delta y_{k+1} \end{pmatrix} \quad \ldots \quad (2.61)$$

Where I is the unit matrix and S and C are positive constants.

2.4 RESULTS

2.4.1 EXAMPLE 1

$$f(x,y) = 2xy - 3$$
$$g(x,y) = x^2 - y - 2$$

METHOD	INITIAL GUESS OF X,Y	OTHER VARIBLES	FINAL SOLUTION	NO. OF ITE.
FIRST ORDER NR METHOD	1.5, 0.9	-	1.698, 0.8334	4
	5, 3	-	1.698, 0.8334	6
	10, 10	-	1.698, 0.8334	7
	-3, -2	-	DIVERGED	-
METHOD 1	1.5, 0.9	-	1.698, 0.8334	4
	5, 3	-	1.698, 0.8334	5
	10, 10	-	1.698, 0.8334	7
	-3, -2	-	1.698, 0.8334	20
METHOD 2	1.5, 0.9	-	1.698, 0.8334	3
	5, 3	-	DIVERGED	-
	10, 10	-	DIVERGED	-
	-3, -2	-	DIVERGED	-
METHOD 3	1.5, 0.9	-	1.698, 0.8334	3
	5, 3	-	1.698, 0.8334	5
	10, 10	-	1.698, 0.8334	5
	-3, -2	-	1.698, 0.8334	16
METHOD 4	1.5, 0.9	α=0.9 β=0.5	1.698, 0.8334	5
	5, 3	α=0.9 β=0.5	1.698, 0.8334	8
	10, 10	α=0.9 β=0.5	1.698, 0.8334	10
	-3, -2	α=0.9 β=0.5	1.698, 0.8334	16
METHOD 5(a)	1.5, 0.9	-	1.698, 0.8334	6
	5, 3	-	1.698, 0.8334	31
	10, 10	-	1.698, 0.8334	98
	-3, -2	-	DIVERGED	-
METHOD 5(b)	1.5, 0.9	-	1.698, 0.8334	4
	5, 3	-	1.698, 0.8334	5
	10, 10	-	1.698, 0.8334	7
	-3, -2	-	1.698, 0.8334	20
METHOD 5(c)	1.5, 0.9	-	1.698, 0.8334	4
	5, 3	-	1.698, 0.8334	30
	10, 10	-	1.698, 0.8334	84
	-3, -2	-	DIVERGED	-
METHOD 5(d)	1.5, 0.9	-	1.698, 0.8334	3
	5, 3	-	1.698, 0.8334	5
	10, 10	-	1.698, 0.8334	5
	-3, -2	-	1.698, 0.8334	16

METHOD	INITIAL GUESS OF X,Y	OTHER VARIBLES	FINAL SOLUTION	NO. OF ITE.
METHOD 6	1.5, 0.9	-	1.698, 0.8334	6
	5, 3	-	DIVERGED	-
	10, 10	-	DIVERGED	-
	-3, -2	-	DIVERGED	-
METHOD 7	1.5, 0.9	-	1.698, 0.8334	8
	5, 3	-	1.698, 0.8334	13
	10, 10	-	1.698, 0.8334	14
	-3, -2	-	1.698, 0.8334	13
DECELERATED CONVERGENCE METHOD	1.5, 0.9	α=1	1.698, 0.8334	3
	5, 3	α=1	1.698, 0.8334	5
	10, 10	α=1	1.698, 0.8334	7
	-3, -2	α=1	DIVERGED	-
TWO-STEP ALGORITHM	1.5, 0.9	α=0.2, C=0.8	1.698, 0.8334	3
	5, 3	α=0.5, C=0.3	1.698, 0.8334	3
	10, 10	α=0.5, C=0.3	1.698, 0.8334	4
	-3, -2	α=0.5, C=0.3	DIVERGED	-
THREE-STEP ALGORITHM	1.5, 0.9	α=0.2, C=0.8	1.698, 0.8334	2
	5, 3	α=0.5, C=0.3	1.698, 0.8334	3
	10, 10	α=0.5, C=0.3	1.698, 0.8334	3
	-3, -2	α=0.5, C=0.3	DIVERGED	-
CONSTANT POSITIVE SHIFT ALGORITHM	1.5, 0.9	S=0.01	1.698, 0.8334	3
	5, 3	S=0.01	1.698, 0.8334	5
	10, 10	S=0.001	1.698, 0.8334	5
	-3, -2	S=0.01	DIVERGED	-
INCREASING POSITIVE SHIFT ALGORITHM	1.5, 0.9	S=0.01,C=0.2	1.698, 0.8334	3
	5, 3	S=0.01,C=0.2	1.698, 0.8334	5
	10, 10	S=0.01,C=0.2	1.698, 0.8334	5
	-3, -2	S=0.01,C=0.2	DIVERGED	-

Table 2.1 Results table for example 1

2.4.2 EXAMPLE 2

$$f(x,y) = x^2 - \cos y$$
$$g(x,y) = \sin x + x^2 + y^3$$

METHOD	INITIAL GUESS OF X,Y	OTHER VARIBLES	FINAL SOLUTION	NO. OF ITE.
FIRST ORDER NR METHOD	1, 1	-	-0.9498, -0.4462	12
	-1, -1	-	-0.9498, -0.4462	5
	10, 10	-	-0.9498, -0.4462	17
	10, -20	-	-0.7069, -1.0475	11
METHOD 1	1, 1	-	-0.7069, -1.0475	7
	-1, -1	-	-0.9498, -0.4462	4
	10, 10	-	-0.7069, -1.0475	9
	10, -20	-	-0.9498, -0.4462	10
METHOD 2	1, 1	-	-0.7069, -1.0475	72
	-1, -1	-	-0.9498, -0.4462	3
	10, 10	-	DIVERGED	-
	10, -20	-	DIVERGED	-
METHOD 3	1, 1	-	-0.7069, -1.0475	6
	-1, -1	-	-0.9498, -0.4462	4
	10, 10	-	-0.9498, -0.4462	9
	10, -20	-	-0.7069, -1.0475	8
METHOD 4	1, 1	α=0.9 β=0.5	-0.7069, -1.0475	9
	-1, -1	α=0.9 β=0.5	-0.9498, -0.4462	6
	10, 10	α=0.9 β=0.5	-0.9498, -0.4462	18
	10, -20	α=0.9 β=0.5	-0.9498, -0.4462	10
METHOD 5(a)	1, 1	-	-0.7069, -1.0475	-
	-1, -1	-	-0.9498, -0.4462	27
	10, 10	-	DIVERGED	-
	10, -20	-	-0.7069, -1.0475	-
METHOD 5(b)	1, 1	-	-0.7069, -1.0475	10
	-1, -1	-	-0.9498, -0.4462	5
	10, 10	-	-0.9498, -0.4462	24
	10, -20	-	-0.9498, -0.4462	12
METHOD 5(c)	1, 1	-	DIVERGED	-
	-1, -1	-	-0.9498, -0.4462	27
	10, 10	-	-0.7069, -1.0475	-
	10, -20	-	DIVERGED	-
METHOD 5(d)	1, 1	-	-0.7069, -1.0475	6
	-1, -1	-	-0.9498, -0.4462	4
	10, 10	-	-0.7069, -1.0475	9
	10, -20	-	-0.9498, -0.4462	8

METHOD	INITIAL GUESS OF X,Y	OTHER VARIBLES	FINAL SOLUTION	NO. OF ITE.
METHOD 6	1, 1	-	DIVERGED	-
	-1, -1	-	-0.9498, -0.4462	10
	10, 10	-	DIVERGED	-
	10, -20	-	DIVERGED	-
METHOD 7	1, 1	-	DIVERGED	-
	-1, -1	-	DIVERGED	-
	10, 10	-	DIVERGED	-
	10, -20	-	DIVERGED	-
DECELERATED CONVERGENCE METHOD	1, 1	α=1	-0.7069, -1.0475	6
	-1, -1	α=1	-0.9498, -0.4462	4
	10, 10	α=1	-0.7069, -1.0475	9
	10, -20	α=1	-0.9498, -0.4462	8
TWO-STEP ALGORITHM	1, 1	α=5, C=5	-0.7069, -1.0475	4
	-1, -1	α=1.5, C=10	-0.9498, -0.4462	3
	10, 10	α=0.5, C=1	-0.7069, -1.0475	5
	10, -20	α=0.5, C=1	-0.9498, -0.4462	5
THREE-STEP ALGORITHM	1, 1	α=0.1, C=5	-0.7069, -1.0475	3
	-1, -1	α=1, C=1	-0.9498, -0.4462	2
	10, 10	α=1, C=1	-0.9498, -0.4462	4
	10, -20	α=0.5, C=1	-0.7069, -1.0475	4
CONSTANT POSITIVE SHIFT ALGORITHM	1, 1	S=0.005	-0.7069, -1.0475	7
	-1, -1	S=0.005	-0.9498, -0.4462	4
	10, 10	S=0.01	-0.9498, -0.4462	13
	10, -20	S=0.005	-0.7069, -1.0475	7
INCREASING POSITIVE SHIFT ALGORITHM	1, 1	S=0.001,C=0.2	-0.7069, -1.0475	6
	-1, -1	S=0.001,C=0.2	-0.9498, -0.4462	4
	10, 10	S=0.001,C=0.2	-0.9498, -0.4462	12
	10, -20	S=0.001,C=0.2	-0.7069, -1.0475	7

Table 2.2 Results table for example 2

2.4.3 EXAMPLE 3

$$f(x,y,z) = e^{(1-x-y-z)} - 1$$
$$g(x,y,z) = x^2 - y$$
$$e(x,y,z) = x^3 - z$$

METHOD	INITIAL GUESS OF X, Y, Z	OTHER VARIBLES	FINAL SOLUTION	NO. OF ITE.
FIRST ORDER NR METHOD	1, 1, 1.2	-	0.5437,0.2956,0.1607	13
	3, 4, 2	-	DIVERGED	-
	10, 10, -10	-	DIVERGED	-
	100, 100, 100	-	DIVERGED	-
METHOD 1	1, 1, 1.2	-	0.5437,0.2956,0.1607	6
	3, 4, 2	-	0.5437,0.2956,0.1607	12
	10, 10, -10	-	0.5437,0.2956,0.1607	13
	100, 100, 100	-	0.5437,0.2956,0.1607	323
METHOD 2	1, 1, 1.2	-	0.5437,0.2956,0.1607	8
	3, 4, 2	-	0.5437,0.2956,0.1607	17
	10, 10, -10	-	DIVERGED	-
	100, 100, 100	-	DIVERGED	-
METHOD 3	1, 1, 1.2	-	0.5437,0.2956,0.1607	6
	3, 4, 2	-	0.5437,0.2956,0.1607	12
	10, 10, -10	-	0.5437,0.2956,0.1607	13
	100, 100, 100	-	0.5437,0.2956,0.1607	303
METHOD 4	1, 1, 1.2	α=0.9 β=0.5	0.5437,0.2956,0.1607	7
	3, 4, 2	α=0.9 β=0.5	0.5437,0.2956,0.1607	13
	10, 10, -10	α=0.9 β=0.5	0.5437,0.2956,0.1607	14
	100, 100, 100	α=0.9 β=0.5	0.5437,0.2956,0.1607	275
METHOD 5(a)	1, 1, 1.2	-	0.5437,0.2956,0.1607	15
	3, 4, 2	-	0.5437,0.2956,0.1607	109
	10, 10, -10	-	0.5437,0.2956,0.1607	-
	100, 100, 100	-	DIVERGED	-
METHOD 5(b)	1, 1, 1.2	-	0.5437,0.2956,0.1607	7
	3, 4, 2	-	0.5437,0.2956,0.1607	31
	10, 10, -10	-	0.5437,0.2956,0.1607	20
	100, 100, 100	-	0.5437,0.2956,0.1607	329
METHOD 5(c)	1, 1, 1.2	-	0.5437,0.2956,0.1607	15
	3, 4, 2	-	0.5437,0.2956,0.1607	109
	10, 10, -10	-	0.5437,0.2956,0.1607	-
	100, 100, 100	-	DIVERGED	-
METHOD 5(d)	1, 1, 1.2	-	0.5437,0.2956,0.1607	6
	3, 4, 2	-	0.5437,0.2956,0.1607	12
	10, 10, -10	-	0.5437,0.2956,0.1607	13
	100, 100, 100	-	0.5437,0.2956,0.1607	303

METHOD	INITIAL GUESS OF X,Y,Z	OTHER VARIBLES	FINAL SOLUTION	NO. OF ITE.
METHOD 6	1, 1, 1.2	-	0.5437,0.2956,0.1607	8
	3, 4, 2	-	0.5437,0.2956,0.1607	14
	10, 10, -10	-	DIVERGED	-
	100, 100, 100	-	DIVERGED	-
METHOD 7	1, 1, 1.2	-	0.5437,0.2956,0.1607	8
	3, 4, 2	-	DIVERGED	-
	10, 10, -10	-	DIVERGED	-
	100, 100, 100	-	DIVERGED	-
DECELERATED CONVERGENCE METHOD	1, 1, 1.2	$\alpha=1$	0.5437,0.2956,0.1607	6
	3, 4, 2	$\alpha=1$	0.5437,0.2956,0.1607	12
	10, 10, -10	$\alpha=1$	0.5437,0.2956,0.1607	13
	100, 100, 100	$\alpha=1$	0.5437,0.2956,0.1607	303
TWO-STEP ALGORITHM	1, 1, 1.2	$\alpha=0.8$, C=0.8	0.5437,0.2956,0.1607	4
	3, 4, 2	$\alpha=0.9$, C=0.4	0.5437,0.2956,0.1607	6
	10, 10, -10	$\alpha=1$, C=0.1	0.5437,0.2956,0.1607	8
	100, 100, 100	$\alpha=1$, C=0.0001	0.5437,0.2956,0.1607	104
THREE-STEP ALGORITHM	1, 1, 1.2	$\alpha=0.9$, C=0.4	0.5437,0.2956,0.1607	4
	3, 4, 2	$\alpha=0.9$, C=0.4	0.5437,0.2956,0.1607	5
	10, 10, -10	$\alpha=1$, C=0.0001	0.5437,0.2956,0.1607	10
	100, 100, 100	$\alpha=1$, C=0.0001	0.5437,0.2956,0.1607	78
CONSTANT POSITIVE SHIFT ALGORITHM	1, 1, 1.2	S=0.005	0.5437,0.2956,0.1607	6
	3, 4, 2	S=0.005	0.5437,0.2956,0.1607	12
	10, 10, -10	S=0.01	0.5437,0.2956,0.1607	13
	100, 100, 100	S=0.005	0.5437,0.2956,0.1607	303
INCREASING POSITIVE SHIFT ALGORITHM	1, 1, 1.2	S=0.005,C=0.2	0.5437,0.2956,0.1607	6
	3, 4, 2	S=0.05,C=0.6	0.5437,0.2956,0.1607	12
	10, 10, -10	S=0.005,C=0.6	0.5437,0.2956,0.1607	13
	100, 100, 100	S=0.005,C=0.2	0.5437,0.2956,0.1607	301

Table 2.3 Results table for example 3

2.4.4 EXAMPLE 4

$$f(x,y,z,w) = x + y + z + w$$
$$g(x,y,z,w) = xy + xz + xw + yz + yw + zw$$
$$e(x,y,z,w) = xyz + xyw + xzw + yzw$$
$$d(x,y,z,w) = xyzw - 4$$

METHOD	INITIAL GUESS OF X,Y,Z,W	OTHER VARIBLES	FINAL SOLUTION	NO. OF ITE.
FIRST ORDER NR METHOD	2+3j,2-3j,4+5j,10-j	-	1+j,1-j,-1+j,-1-j	10
	10+10j,-5+7j,-20+j,4-6j	-	1+j,-1+j,-1-j,1-j	12
	1+j,5,7,10-20j	-	1+j,1-j,-1+j,-1-j	11
	50-75j,-100+50j,20-80j,200	-	-1-j,-1+j,1-j,1+j	20
METHOD 1	2+3j,2-3j,4+5j,10-j	-	1-j,-1-j,-1+j,1+j	13
	10+10j,-5+7j,-20+j,4-6j	-	-1+j,-1-j,1-j,1+j	14
	1+j,5,7,10-20j	-	1+j,-1+j,1-j,-1-j	14
	50-75j,-100+50j,20-80j,200	-	-1+j,1+j,-1-j,1-j	27
METHOD 2	2+3j,2-3j,4+5j,10-j	-	-1+j,-1-j,1+j,1-j	13
	10+10j,-5+7j,-20+j,4-6j	-	DIVERGED	-
	1+j,5,7,10-20j	-	1+j,-1-j,1-j,-1+j	10
	50-75j,-100+50j,20-80j,200	-	DIVERGED	-
METHOD 3	2+3j,2-3j,4+5j,10-j	-	1-j,-1-j,-1+j,1+j	7
	10+10j,-5+7j,-20+j,4-6j	-	1+j,-1+j,-1-j,1-j	8
	1+j,5,7,10-20j	-	1+j,1-j,-1+j,-1-j	7
	50-75j,-100+50j,20-80j,200	-	-1-j,-1+j,1-j,1+j	12
METHOD 4	2+3j,2-3j,4+5j,10-j	α=0.9 β=0.6	1+j,1-j,-1+j,1+j	8
	10+10j,-5+7j,-20+j,4-6j	α=0.9 β=0.6	1+j,-1+j,-1-j,-1-j	10
	1+j,5,7,10-20j	α=0.9 β=0.6	1+j,1-j,-1+j,-1-j	10
	50-75j,-100+50j,20-80j,200	α=0.9 β=0.6	-1-j,-1+j,1-j,1+j	12
METHOD 5(a)	2+3j,2-3j,4+5j,10-j	-	DIVERGED	-
	10+10j,-5+7j,-20+j,4-6j	-	DIVERGED	-
	1+j,5,7,10-20j	-	DIVERGED	-
	50-75j,-100+50j,20-80j,200	-	DIVERGED	-
METHOD 5(b)	2+3j,2-3j,4+5j,10-j	-	-1+j,-1-j,1+j,1-j	31
	10+10j,-5+7j,-20+j,4-6j	-	DIVERGED	-
	1+j,5,7,10-20j	-	1+j,-1-j,1-j,-1+j	40
	50-75j,-100+50j,20-80j,200	-	DIVERGED	-
METHOD 5(c)	2+3j,2-3j,4+5j,10-j	-	DIVERGED	-
	10+10j,-5+7j,-20+j,4-6j	-	DIVERGED	-
	1+j,5,7,10-20j	-	DIVERGED	-
	50-75j,-100+50j,20-80j,200	-	DIVERGED	-
METHOD 5(d)	2+3j,2-3j,4+5j,10-j	-	1-j,-1-j,-1+j,1+j	13
	10+10j,-5+7j,-20+j,4-6j	-	1+j,-1+j,-1-j,1-j	14
	1+j,5,7,10-20j	-	1+j,1-j,-1+j,-1-j	14
	50-75j,-100+50j,20-80j,200	-	-1-j,-1+j,1-j,1+j	27

METHOD	INITIAL GUESS OF X,Y,Z,W	OTHER VARIBLES	FINAL SOLUTION	NO. OF ITE.
METHOD 6	2+3j,2-3j,4+5j,10-j	-	DIVERGED	-
	10+10j,-5+7j,-20+j,4-6j	-	DIVERGED	
	1+j,5,7,10-20j	-	DIVERGED	-
	50-75j,-100+50j,20-80j,200	-	DIVERGED	-
METHOD 7	2+3j,2-3j,4+5j,10-j	-	1+j,1-j,-1+j,-1-j	40
	10+10j,-5+7j,-20+j,4-6j	-	1+j,-1+j,-1-j,1-j	18
	1+j,5,7,10-20j	-	-1+j,1-j,1+j,-1-j	19
	50-75j,-100+50j,20-80j,200	-	-1-j,-1+j,1-j,1+j	30
DECELERATED CONVERGENCE METHOD	2+3j,2-3j,4+5j,10-j	α=1	1-j,-1-j,-1+j,1+j	7
	10+10j,-5+7j,-20+j,4-6j	α=1	1+j,-1+j,-1-j,1-j	8
	1+j,5,7,10-20j	α=1	1+j,1-j,-1+j,-1-j	7
	50-75j,-100+50j,20-80j,200	α=1	-1-j,-1+j,1-j,1+j	12
TWO-STEP ALGORITHM	2+3j,2-3j,4+5j,10-j	α=1, C=0.0001	1+j,1-j,-1-j,-1-j	5
	10+10j,-5+7j,-20+j,4-6j	α=1, C=0.0001	1+j,-1+j,-1-j,1-j	5
	1+j,5,7,10-20j	α=0.9, C=0.5	1+j,1-j,-1+j,-1-j	5
	50-75j,-100+50j,20-80j,200	α=1, C=0.0001	-1+j,1+j,-1-j,1-j	13
THREE-STEP ALGORITHM	2+3j,2-3j,4+5j,10-j	α=1, C=0.0001	1+j,1-j,-1-j,-1-j	4
	10+10j,-5+7j,-20+j,4-6j	α=0.9, C=0.5	1+j,-1+j,-1-j,1-j	4
	1+j,5,7,10-20j	α=1, C=0.0001	1+j,1-j,-1+j,-1-j	4
	50-75j,-100+50j,20-80j,200	α=1, C=0.0001	-1+j,1+j,-1-j,1-j	9
CONSTANT POSITIVE SHIFT ALGORITHM	2+3j,2-3j,4+5j,10-j	S=0.005	1+j,1-j,-1-j,-1-j	7
	10+10j,-5+7j,-20+j,4-6j	S=0.005	1+j,-1+j,-1-j,1-j	7
	1+j,5,7,10-20j	S=0.001	1+j,1-j,-1+j,-1-j	7
	50-75j,-100+50j,20-80j,200	S=0.005	-1-j,-1+j,1-j,1+j	11
INCREASING POSITIVE SHIFT ALGORITHM	2+3j,2-3j,4+5j,10-j	S=0.005,C=0.2	1+j,1-j,-1-j,-1-j	7
	10+10j,-5+7j,-20+j,4-6j	S=0.001,C=0.2	1+j,-1+j,-1-j,1-j	7
	1+j,5,7,10-20j	S=0.005,C=0.2	1+j,1-j,-1+j,-1-j	7
	50-75j,-100+50j,20-80j,200	S=0.005,C=0.2	-1-j,1-j,1-j,1+j	10

Table 2.4 Results table for example 4

CHAPTER 3

NEWTON-RAPHSON METHOD FOR LOAD-FLOW SOLUTION

3.1 INTRODUCTION

Load flow solution is a solution of the network under steady state condition subjected to certain inequality constraints under which the system operates [7,8]. These constraints can be in the form of load nodal voltages, reactive power generation of the generators, the tap settings of a tap changing under load transformer etc.

The load flow solution gives the nodal voltages and phase angles and hence the power injection at all the buses and power flows through interconnecting transmission lines. Load flow solution is essential for designing a new power system and for planning extension of the existing one for increased load demand. These analyses require the calculation of numerous load flows under both normal and abnormal (outage of transmission line, or outage of some generating source) operating conditions. Load flow solution also gives the initial conditions of the system when the transient behavior of the system is to be studied. Single line representation is enough under balanced operating conditions. A load flow solution of the power system requires mainly the following steps:

- Formulation of the network equations.
- Suitable mathematical technique for solution of the equations.

Under steady state condition the network equations will be in the form of simple algebraic equations. The load and hence generation are continually changing in a real power system, but for solving load flow it is assumed that loads and hence generation are fixed at a particular value over a suitable period of time. E.g. half an hour or so.

In a power system each bus or node is associated with four quantities, real and reactive powers, bus voltages magnitude and its phase angle. In a load flow solution two out of four quantities are specified and the remaining two are required to be obtained through the solution of the equations. The buses are classified depending upon the quantities specified into the following three categories

Load bus: At this bus the real and reactive components of power are specified. It is desired to find out the voltage magnitude V and phase angle δ through the load flow solution. Voltage at load bus can be allowed to vary within the permissible value e.g. 5%.

Generator bus or voltage controlled bus: Here the voltage magnitude corresponding to the generation voltage and real power P_G corresponding to its ratings are specified. It is required to find out the reactive power generation Q_G and the phase angle δ of the bus voltage.

Slack, swing or reference bus: Here the voltage magnitude V and phase angle δ are specified. This will take care of the additional power generation required and Transmission losses. It is required to find of real and reactive power generations (P_G, Q_G) at this bus.

Load flow solution can be achieved with iterative methods. There are many kinds of iterative methods out of which Newton-Raphson method is superior. Newton-Raphson method is illustrated in the following sections.

First order Newton-Raphson method has some disadvantages like they may fail to converge in ill conditioned systems like systems having large R/X ratio, and when the Jacobian matrix becomes singular in this condition the modified methods which are described in section (3.3) are useful the Hessian term which will be added to Jacobian will make it non-singular and solution can be achieved.

3.2 NEWTON-RAPHSON LOAD FLOW (NRLF) METHOD

For a N-bus power system there will be n equations for real power P_i and n-equation for reactive power Q_i.

$$P_i = P_{Gi} - P_{Di} = \sum_{j=1}^{n} \left[V_i \, V_j \, Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij}) \right]$$

$$Q_i = Q_{Gi} - Q_{Di} = \sum_{j=1}^{n} \left[V_i \, V_j \, Y_{ij} \sin(\delta_i - \delta_j - \theta_{ij}) \right] \quad \ldots \; (3.1)$$

$$i = 1,2,3,\ldots\ldots,n$$

The number of equations to be solved depends upon the specifications we have. If the total number of buses is n and number of generator buses is m then the number equation to be solved will be number of known P_i's and number of known Q_i's. In the above conditions number of known P_i's are n-1 and the number of known Q_i's are n-m, therefore the total number of simultaneous equation will be 2*n-m-1, and number of unknown quantities also 2*n-m-1. Unknowns to be find out are δ at all the buses except slack (i.e. n-1) and V at load bus (i.e. n-m). Following the method explained in section (2.1.2) problem can be formulated as

$$\begin{pmatrix} \Delta P \\ \Delta Q \end{pmatrix} = \begin{pmatrix} \dfrac{\partial P}{\partial \delta} & \dfrac{\partial P}{\partial V} \\ \dfrac{\partial Q}{\partial \delta} & \dfrac{\partial Q}{\partial V} \end{pmatrix} \begin{pmatrix} \Delta \delta \\ \Delta V \end{pmatrix} \quad \ldots \; (3.2)$$

Real power terms will be calculated for all the buses except slack and reactive power terms will be calculated for load buses. In the above equation

$\begin{pmatrix} \Delta P \\ \Delta Q \end{pmatrix}$ is the mismatch vector

$\begin{pmatrix} \Delta \delta \\ \Delta V \end{pmatrix}$ is the correction vector

and

$$J = \begin{pmatrix} \frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial V} \end{pmatrix} \text{ is the Jacobian matrix } \ldots \quad (3.3)$$

The elements of the Jacobian matrix can be calculated using the following equations

$$\begin{aligned}
\frac{\partial P_i}{\partial \delta_i} &= -Q_i - V_i^2\, B_{ii} \\
\frac{\partial P_i}{\partial \delta_j} &= V_i\, V_j\, Y_{ij}\, \sin(\delta_i - \delta_j - \theta_{ij}) \\
\frac{\partial P_i}{\partial V_i} &= \sum_{j=1}^{n} V_j\, Y_{ij}\, \cos(\delta_i - \delta_j - \theta_{ij}) + V_i\, G_{ii} \\
\frac{\partial P_i}{\partial V_j} &= V_i\, Y_{ij}\, \cos(\delta_i - \delta_j - \theta_{ij}) \qquad \ldots \quad (3.4) \\
\frac{\partial Q_i}{\partial \delta_i} &= P_i - V_i^2\, G_{ii} \\
\frac{\partial Q_i}{\partial \delta_j} &= -V_i\, V_j\, Y_{ij}\, \cos(\delta_i - \delta_j - \theta_{ij}) \\
\frac{\partial Q_i}{\partial V_i} &= \sum_{j=1}^{n} V_j\, Y_{ij}\, \sin(\delta_i - \delta_j - \theta_{ij}) + V_i\, B_{ii} \\
\frac{\partial Q_i}{\partial V_j} &= V_i\, Y_{ij}\, \sin(\delta_i - \delta_j - \theta_{ij})
\end{aligned}$$

Procedure for this iterative method is same as that explained in section (2.1.2) except some adjustments which are explained in section (3.2.1) with those adjustments NRLF method will be carried out and voltages and angles will be found out. With these values line flow and all required information can be obtained.

3.2.1 ADJUSTMENTS IN NRLF SOLUTION

Some constraints are applied on Load flow solution in the present thesis work, which is not the case in general problems explained in previous chapter. The solution has to have some limits such as voltage, and reactive power should not be greater/lesser than its limits. So some modifications are made to overcome this problem and they are explained below

3.2.1.1 HANDLING OF Q-LIMIT AT GENERATOR BUSES

Since Q_i at generator buses is not given, it is calculated in each iteration at all generator buses and it is been checked for the condition

$$Q_{i\,\min} < Q_i < Q_{i\,\max} \quad \ldots (3.5)$$

If the above mentioned condition satisfied then the ith bus will remain as generator bus and there will be no change in the procedure.

If the condition violates then Q_i will be set at the limit values it can be either lower limit or upper limit depending upon the violation and i^{th} bus will be treated as load bus from the next iteration. Because of this change of bus type one additional equation corresponding to ΔQ_i will be added to NRLF scheme.

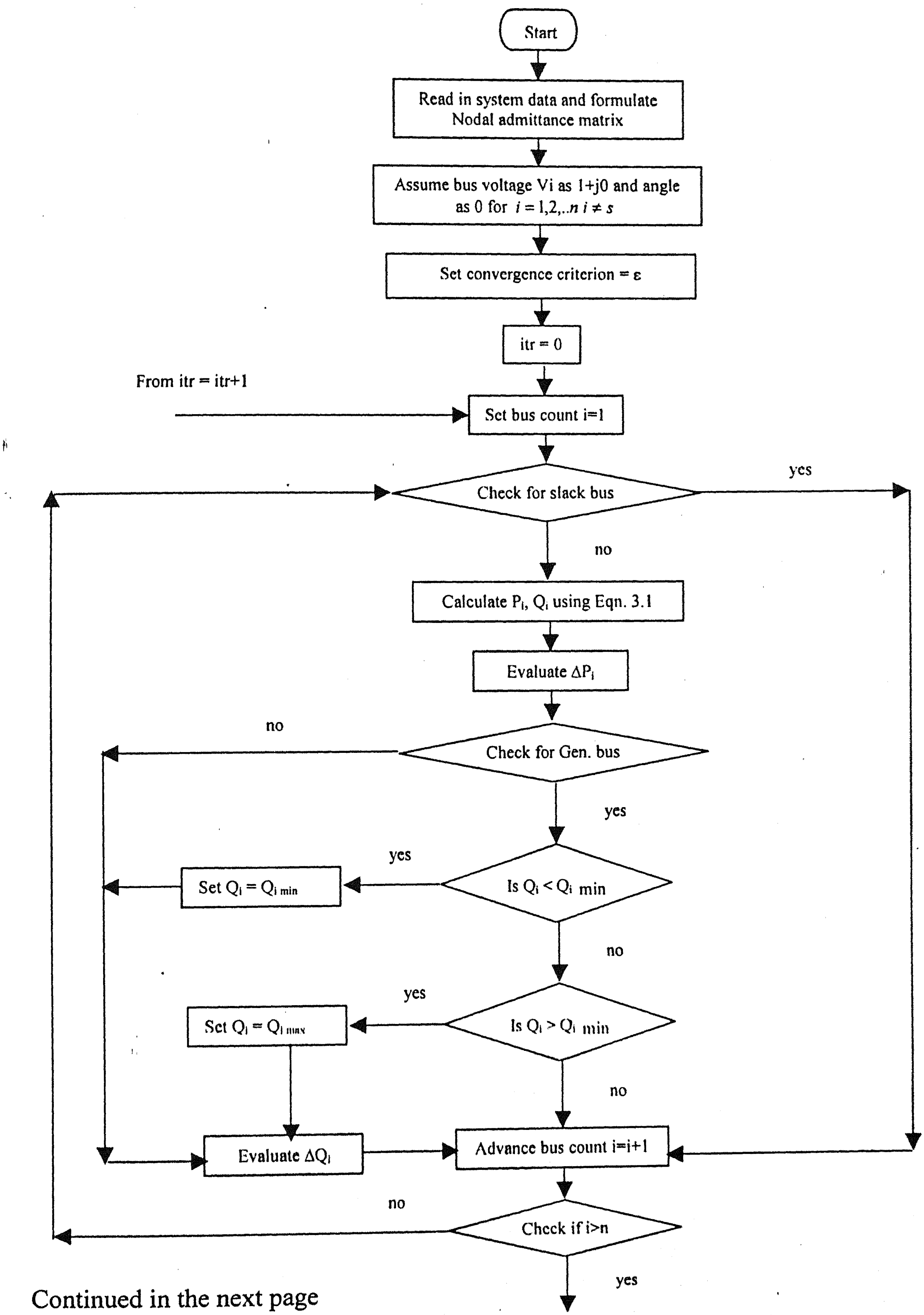
3.2.1.2 HANDLING OF V-LIMIT AT LOAD BUSES:

After each iteration voltages and angles are updated with the obtained correction matrix. Voltage has to satisfy the conditioned given below

$$V_{i\,\min} < V_i < V_{i\,\max} \quad \ldots \quad (3.6)$$

If it satisfies the condition then there will be no change in the process, if it violates then voltage has to be fixed at one of its lower or upper limits depending upon the violation and the bus type is been changed to generator type. Because of this one equation corresponding to ΔQ_i will be deleted from NRLF scheme, so the size of Jacobian matrix will be decreased.

3.2.2 FLOW CHART



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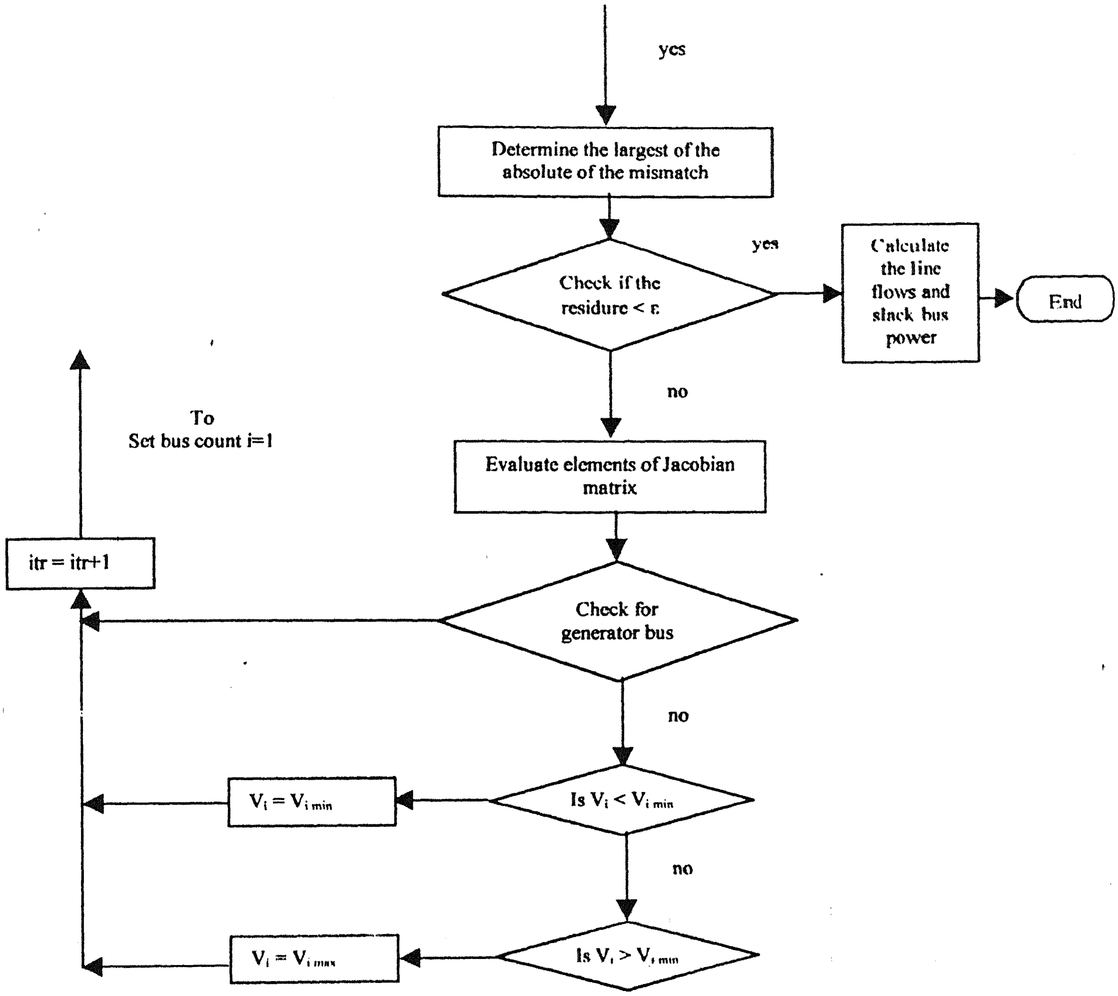


Fig 3.1(a) Flow chart for load flow solution using Newton-Raphson method

3.3 MODIFIED NRLF PROBLEM

Methods which are explained in sections (2.2, 2.3) can be applied to load flow problem. Procedure for these methods is same as explained in those sections except that the elements of Hessian matrix will be different, Hessian matrix for this method will be

$$H = \begin{pmatrix} \frac{\partial^2 P_i}{\partial \delta_i^2} & \cdots & \frac{\partial^2 P_i}{\partial \delta_k \partial \delta_j} & \cdots & \frac{\partial^2 P_i}{\partial V_i \partial \delta_i} & \cdots & \frac{\partial^2 P_i}{\partial V_k \partial \delta_j} & \cdots & \frac{\partial^2 P_i}{\partial V_i^2} & \cdots & \frac{\partial^2 P_i}{\partial V_k \partial V_j} \\ \frac{\partial^2 Q_i}{\partial \delta_i^2} & \cdots & \frac{\partial^2 Q_i}{\partial \delta_k \partial \delta_j} & \cdots & \frac{\partial^2 Q_i}{\partial V_i \partial \delta_i} & \cdots & \frac{\partial^2 Q_i}{\partial V_k \partial \delta_j} & \cdots & \frac{\partial^2 Q_i}{\partial V_i^2} & \cdots & \frac{\partial^2 Q_i}{\partial V_k \partial V_j} \end{pmatrix} \quad \ldots (3.7)$$

$$\begin{aligned} &Let \\ &e_i = V_i \cos(\delta_i) \\ &f_i = V_i \sin(\delta_i) \\ &a_{ij} = G_{ij}\, e_j - B_{ij}\, f_j \\ &b_{ij} = G_{ij}\, f_j + B_{ij}\, e_j \\ &Y_{ij} = G_{ij} + j\, B_{ij} \end{aligned} \quad \cdots (3.8)$$

Hessian terms can be calculated using the formulae given below

$$\begin{aligned} &\frac{\partial^2 P_i}{\partial \delta_i^2} = -Pcal_i + V_i^2\, G_{ii} \\ &\frac{\partial^2 P_i}{\partial \delta_k \partial \delta_j} = -a_{ij} e_i - b_{ij} f_i \quad for\ j = k,\ j \neq i,\ k \neq i \\ &\frac{\partial^2 P_i}{\partial \delta_k \partial \delta_j} = 0 \quad for\ j \neq k,\ k \neq i \\ &\frac{\partial^2 P_i}{\partial \delta_k \partial \delta_j} = a_{ik} e_i + b_{ik} f_i \quad for\ j = i,\ k \neq i \\ &\frac{\partial^2 P_i}{\partial \delta_k \partial \delta_j} = a_{ij} e_i + b_{ij} f_i \quad for\ k = i,\ j \neq i \end{aligned} \quad \ldots (3.9)$$

$$\frac{\partial^2 P_i}{\partial V_i \partial \delta_i} = -Qcal_i - V_i^2 B_{ii}$$

$$\frac{\partial^2 P_i}{\partial V_k \partial \delta_j} = a_{ij} f_i - b_{ij} e_i \quad for\ j = k,\ j \neq i$$

$$\frac{\partial^2 P_i}{\partial V_k \partial \delta_j} = 0 \quad for\ j \neq k,\ k \neq i \qquad \ldots (3.10)$$

$$\frac{\partial^2 P_i}{\partial V_k \partial \delta_j} = -a_{ik} f_i + b_{ik} e_i \quad for\ j = i,\ k \neq i$$

$$\frac{\partial^2 P_i}{\partial V_k \partial \delta_j} = a_{ij} f_i - b_{ij} e_i \quad for\ k = i,\ j \neq i$$

$$\frac{\partial^2 P_i}{\partial V_i^2} = 2 V_i^2 G_{ii}$$

$$\frac{\partial^2 P_i}{\partial V_k \partial V_j} = 0 \quad for\ j = k,\ j \neq i$$

$$\frac{\partial^2 P_i}{\partial V_k \partial V_j} = 0 \quad for\ j \neq k,\ k \neq i \qquad \ldots (3.11)$$

$$\frac{\partial^2 P_i}{\partial V_k \partial V_j} = a_{ik} e_i + b_{ik} f_i \quad for\ j = i,\ k \neq i$$

$$\frac{\partial^2 P_i}{\partial V_k \partial V_j} = a_{ij} e_i + b_{ij} f_i \quad for\ k = i,\ j \neq i$$

$$\frac{\partial^2 Q_i}{\partial \delta_i^2} = -Qcal_i - V_i^2 B_{ii}$$

$$\frac{\partial^2 Q_i}{\partial \delta_k \partial \delta_j} = -a_{ij} f_i + b_{ij} e_i \quad for\ j = k,\ j \neq i,\ k \neq i$$

$$\frac{\partial^2 Q_i}{\partial \delta_k \partial \delta_j} = 0 \quad for\ j \neq k,\ k \neq i \qquad \ldots (3.12)$$

$$\frac{\partial^2 Q_i}{\partial \delta_k \partial \delta_j} = a_{ik} f_i - b_{ik} e_i \quad for\ j = i,\ k \neq i$$

$$\frac{\partial^2 Q_i}{\partial \delta_k \partial \delta_j} = a_{ij} f_i - b_{ij} e_i \quad for\ k = i,\ j \neq i$$

$$\frac{\partial^2 Q_i}{\partial V_i \partial \delta_i} = Pcal_i - V_i^2 G_{ii}$$

$$\frac{\partial^2 Q_i}{\partial V_k \partial \delta_j} = -a_{ij} e_i - b_{ij} f_i \quad for\ j = k,\ j \neq i$$

$$\frac{\partial^2 Q_i}{\partial V_k \partial \delta_j} = 0 \quad for\ j \neq k,\ k \neq i \qquad \ldots \quad (3.13)$$

$$\frac{\partial^2 Q_i}{\partial V_k \partial \delta_j} = a_{ik} e_i + b_{ik} f_i \quad for\ j = i,\ k \neq i$$

$$\frac{\partial^2 Q_i}{\partial V_k \partial \delta_j} = -a_{ij} e_i - b_{ij} f_i \quad for\ k = i,\ j \neq i$$

$$\frac{\partial^2 Q_i}{\partial V_i^2} = -2 V_i^2 B_{ii}$$

$$\frac{\partial^2 Q_i}{\partial V_k \partial V_j} = 0 \quad for\ j = k,\ j \neq i$$

$$\frac{\partial^2 Q_i}{\partial V_k \partial V_j} = 0 \quad for\ j \neq k,\ k \neq i \qquad \ldots \quad (3.14)$$

$$\frac{\partial^2 Q_i}{\partial V_k \partial V_j} = a_{ik} f_i - b_{ik} e_i \quad for\ j = i,\ k \neq i$$

$$\frac{\partial^2 Q_i}{\partial V_k \partial V_j} = a_{ij} f_i - b_{ij} e_i \quad for\ k = i,\ j \neq i$$

If total number of buses is n and total number of generator buses is m including slack bus the size of Hessian matrix will be $(2n - m - 1\ X\ 3n^2 + m^2 - 3nm - 3n + m + 1)$.

Here also the correction matrix will be divided into two matrices one is W matrix and the other will be first order correction matrix. W matrix here will look like

$$W = \begin{bmatrix} W_1 & W_2 \\ W_3 & W_4 \\ W_5 & W_6 \end{bmatrix} \qquad \ldots \quad (3.15)$$

Sizes of these sub matrices will be

$W_1 \rightarrow (n^2\text{-}2n\text{+}1)\ X\ (n\text{-}1)$

$W_2 \rightarrow (n^2\text{-}2n\text{+}1)\ X\ (n\text{-}m)$ ... (3.16)

$W_3 \rightarrow (n^2\text{-}nm\text{-}n\text{+}m)\ X\ (n\text{-}1)$

$W_4 \rightarrow (n^2\text{-}nm\text{-}n\text{+}m) \; X \; (n\text{-}m)$

$W_5 \rightarrow (n^2\text{+}m^2\text{-}2nm) \; X \; (n\text{-}1)$ … (3.16)

$W_6 \rightarrow (n^2\text{+}m^2\text{-}2nm) \; X \; (n\text{-}m)$

These matrix elements will be

$$W_1 = \begin{pmatrix} \Delta\delta_2 & 0 & \dots & \dots & 0 \\ 0 & \Delta\delta_2 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \dots & \Delta\delta_2 \\ \dots & \dots & \dots & \dots & \dots \\ \Delta\delta_n & 0 & \dots & \dots & 0 \\ 0 & \Delta\delta_n & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \dots & \Delta\delta_n \end{pmatrix} \quad \dots \quad (3.17)$$

$$W_2 = \begin{pmatrix} 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 \end{pmatrix} \quad \dots \quad (3.18)$$

$$W_3 = \begin{pmatrix} \Delta V_2 & 0 & \dots & \dots & 0 \\ 0 & \Delta V_2 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \dots & \Delta V_2 \\ \dots & \dots & \dots & \dots & \dots \\ \Delta V_n & 0 & \dots & \dots & 0 \\ 0 & \Delta V_n & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \dots & \Delta V_n \end{pmatrix} \quad \dots \quad (3.19)$$

$$W_5 = \begin{pmatrix} 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 \end{pmatrix} \quad \dots \quad (3.20)$$

$$W_4 = \begin{pmatrix} \Delta\delta_2 & 0 & \dots & \dots & 0 \\ 0 & \Delta\delta_2 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \dots & \Delta\delta_2 \\ \dots & \dots & \dots & \dots & \dots \\ \Delta\delta_n & 0 & \dots & \dots & 0 \\ 0 & \Delta\delta_n & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \dots & \Delta\delta_n \end{pmatrix} \quad \dots (3.21)$$

$$W_6 = \begin{pmatrix} \Delta V_2 & 0 & \dots & \dots & 0 \\ 0 & \Delta V_2 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \dots & \Delta V_2 \\ \dots & \dots & \dots & \dots & \dots \\ \Delta V_n & 0 & \dots & \dots & 0 \\ 0 & \Delta V_n & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \dots & \Delta V_n \end{pmatrix} \quad \dots (3.22)$$

In sub matrix W_6 voltage corrections of load buses will only come, and that of generator buses will not be appear. Here bus number 1 is considered as slack bus that is why elements in all the matrices started from 2.

These matrices will be used in the methods explained in sections (2.2 & 2.3). Procedure will be same as explained in these sections. Results are included at the end of this chapter. Results are shown for 14-bus system, 13-bus system and 11-bus system. In case of 13-bus system resistance values are increased 20 times to make R/X ratio greater than one in which case first order Newton-Raphson method diverges. Proposed methods got converged in this case also. Data for these systems and system diagrams are included in the appendix A.

3.4 RESULTS

3.4.1 13-BUS SYSTEM

Results are tabulated here for both normal case and Ill conditioned case. Ill conditioning has created by increasing R/X ratio. In table 3.1 final voltage and angle values are given and in table 3.2 comparison has given between different methods. Solution started with flat start. Data for this system is given in appendix A. Results are shown graphically in Fig 3.1. Fig A.1 shows the 13 bus system. Table A.1 gives bus data, Table A.2 gives line data and Table A.3 gives Transformer data.

Case 1: With given line data

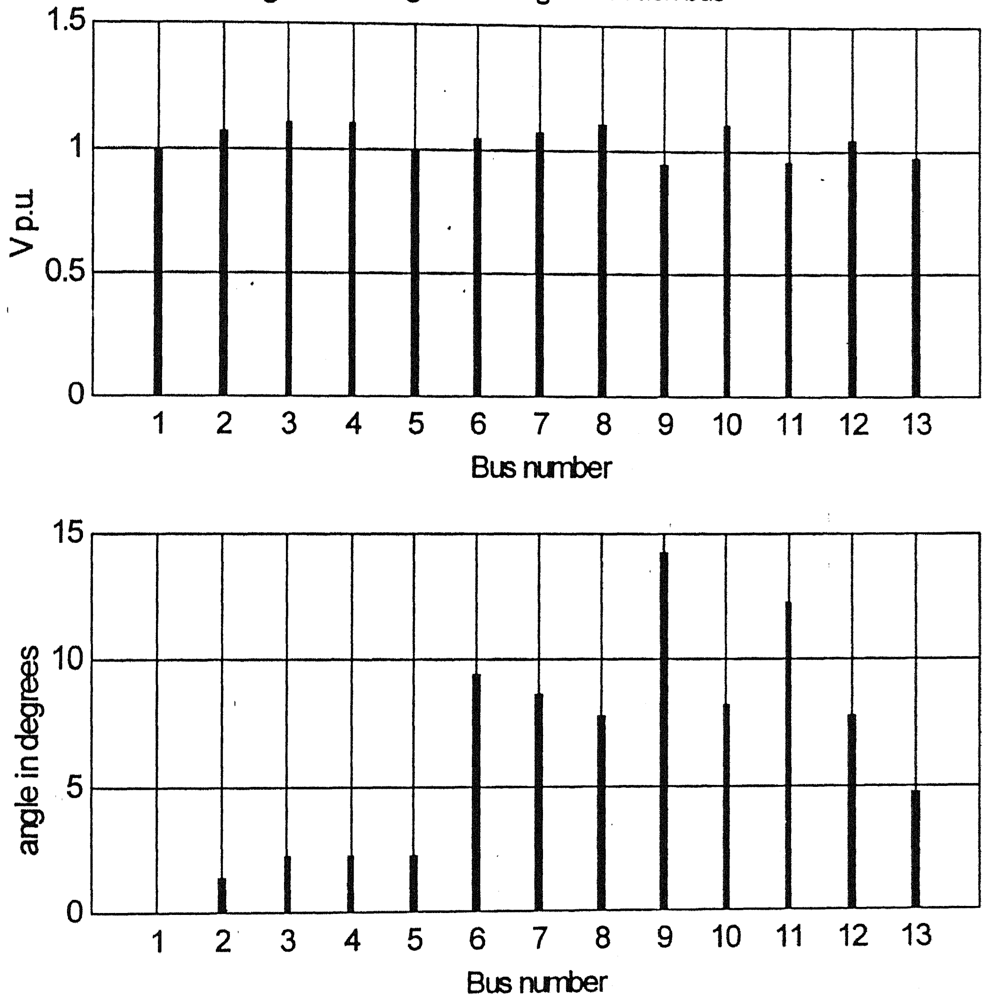
Table 3.1 Load Flow Solution for 13 bus system

Bus Voltage (pu)	Phase Angle (deg.)
1.000000	0.000000
1.063717	1.347255
1.179639	1.945324
1.160323	1.984012
1.000000	2.110602
1.037000	9.211764
1.063684	8.365741
1.100000	7.426077
0.943000	13.840178
1.100000	7.826597
0.957134	11.835802
1.036184	7.363695
0.969457	4.326548

Table 3.2 Comparison of different methods for 13 bus system

METHOD	OTHER VARIABLES	ITERATIONS
FIRST ORDER	--	4
METHOD 1	--	3
METHOD 3	--	4
METHOD 4	α=1.0 β=0.95	4
METHOD 5(a)	--	8
METHOD 5(b)	--	4
METHOD 5(c)	--	9
METHOD 6	--	4
DECELERATED CONVERGENCE METHOD	α=1	3
TWO-STEP ALGORITHM	α=0.6, C=0.001	3
THREE-STEP ALGORITHM	α=0.6, C=0.0001	2
CONSTANT POSITIVE SHIFT ALGORITHM	S=0.005	4
INCREASING POSITIVE SHIFT ALGORITHM	S=0.005,C=0.2	4

Fig 3.1 voltages and angles at each bus



Case 2: By increasing Resistance value 20 times to make R/X ratio greater than 1. Final voltages and angles at all the buses are shown in table 3.3, and comparison between different methods is shown in table 3.4. Results are graphically shown in Fig 3.2. Flat start has been considered here and the convergence criterion is all the elements of mismatch vector have to become less than 0.0001 p.u..

Table 3.3 Load flow solution for 13 bus system with R/X>1

Bus Voltage (pu)	**Phase Angle (deg.)**
1.000000	0.000000
1.062635	-0.354826
1.100000	0.585540
1.100000	0.585522
1.000000	0.601971
1.037000	11.330992
1.029016	4.736914
1.100000	2.420004
0.943000	35.536359
1.100000	17.938702
0.800000	19.881649
1.016978	5.376403
1.100000	4.009542

Table 3.4 Comparison of different methods for 13 bus system with R/X > 1

METHOD	OTHER VARIABLES	ITERATIONS
FIRST ORDER	--	DIVERGED
METHOD 1	--	9
METHOD 3	--	5
METHOD 4	α=1.0 β=0.95	5
METHOD 5(a)	--	DIVERGED
METHOD 5(b)	--	6
METHOD 5(c)	--	9
METHOD 6	--	7
DECELERATED CONVERGENCE METHOD	α=1	5
TWO-STEP ALGORITHM	α=0.6, C=0.001	4
THREE-STEP ALGORITHM	α=0.6, C=0.0001	2
CONSTANT POSITIVE SHIFT ALGORITHM	S=0.0005	5
INCREASING POSITIVE SHIFT ALGORITHM	S=0.0005,C=0.2	5

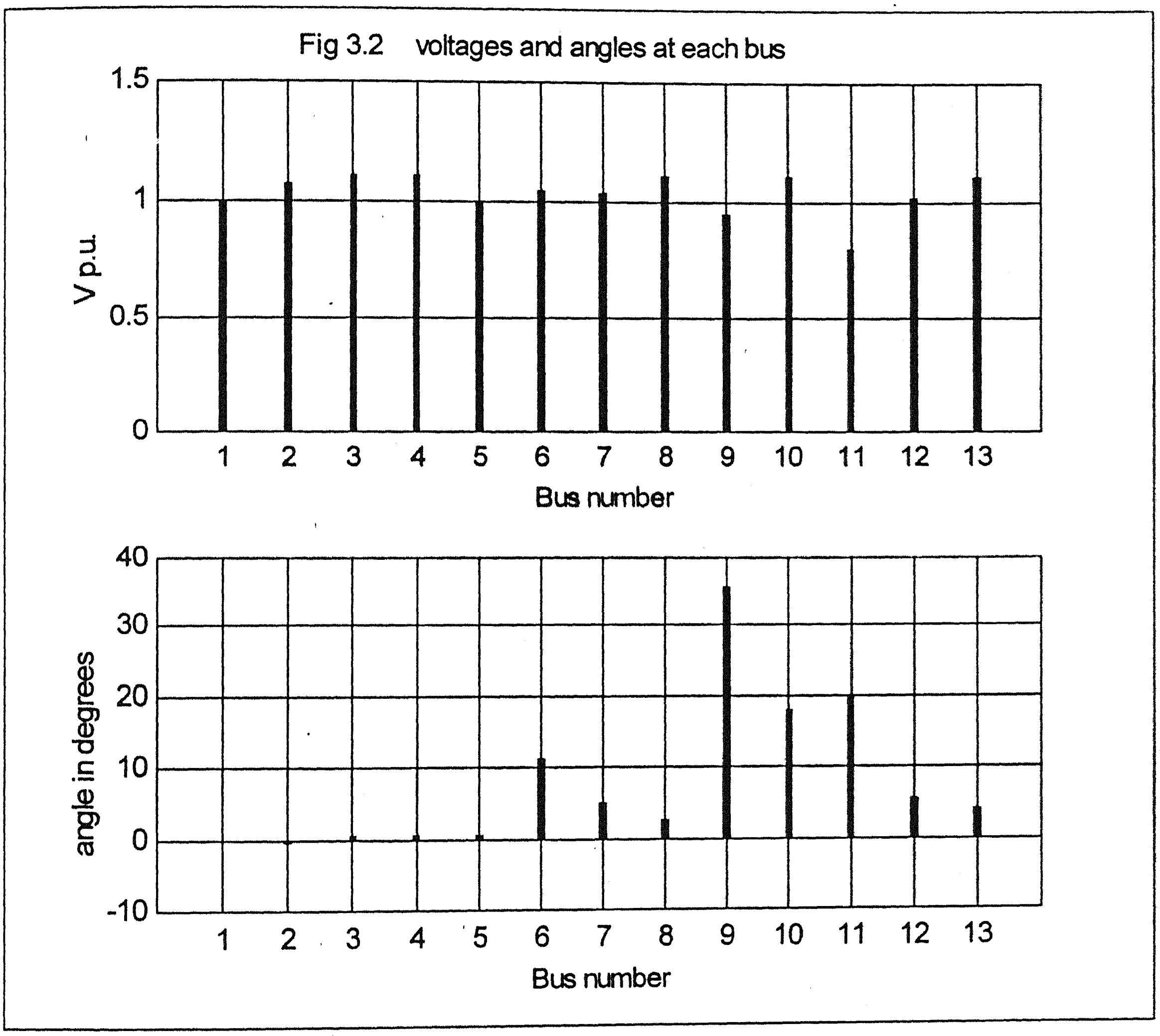


Fig 3.2 voltages and angles at each bus
V p.u.
1.5
1
0.5
0
1 2 3 4 5 6 7 8 9 10 11 12 13
Bus number
angle in degrees
40
30
20
10
0
-10
1 2 3 4 5 6 7 8 9 10 11 12 13
Bus number

3.4.2 11-BUS SYSTEM

Data for this system is given in appendix A. Convergence criterion is all the elements of mismatch vector has to become less than 0.0001 p.u.. Results are given below. Table 3.5 gives the final voltages and phase angles at all the buses and table 3.6 gives the comparison between different methods. Final voltages and angles are shown graphically in Fig 3.3. Fig A.2 shows the system. Table A.4 gives bus data and Table A.5 gives Y bus matrix.

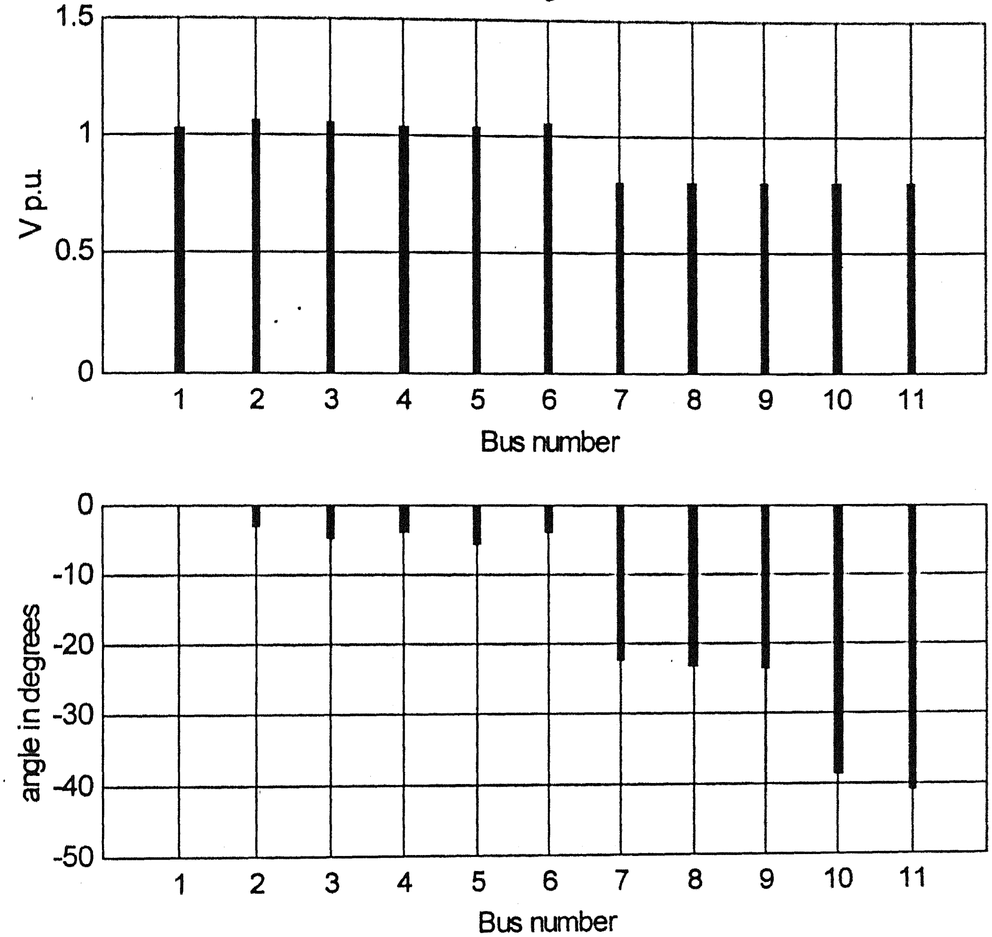
Table 3.5 Load Flow solution for 11 bus system

Bus Voltage (pu)	**Phase Angle (deg.)**
1.024000	0.000000
1.059109	-2.881015
1.048164	-4.635268
1.031116	-3.644625
1.035652	-5.451005
1.050978	-3.728565
0.800000	-22.477944
0.800000	-23.175930
0.800000	-23.451258
0.800000	-38.527317
0.800000	-40.919689

Table 3.6 Comparison of different methods for 11 bus system

METHOD	OTHER VARIABLES	ITERATIONS
FIRST ORDER	--	5
METHOD 1	--	5
METHOD 3	--	4
METHOD 4	α=1.0 β=0.95	4
METHOD 5(a)	--	DIVERGED
METHOD 5(b)	--	DIVERGED
METHOD 5(c)	--	DIVERGED
METHOD 6	--	5
DECELERATED CONVERGENCE METHOD	α=1	4
TWO-STEP ALGORITHM	α=0.6, C=0.001	2
THREE-STEP ALGORITHM	α=0.6, C=0.0001	2
CONSTANT POSITIVE SHIFT ALGORITHM	S=0.0005	5
INCREASING POSITIVE SHIFT ALGORITHM	S=0.0005,C=0.2	4

Fig 3.3 voltages and angles at each bus



3.4.3 14-BUS SYSTEM

Data for this system is given in appendix A. Convergence criterion is all the elements of mismatch vector has to become less than 0.0001 p.u.. Results are given below. Table 3.7 gives the final voltages and phase angles at all the buses and table 3.8 gives the comparison between different methods. Final voltages and angles are shown graphically in Fig 3.4. Fig A.3 shows the system. Table A.6 gives Generator bus voltage, Table A.7 gives Generator data, Table A.8 gives Transformer data, Table A.9 gives Load bus data and Table A.10 gives line data.

Table 3.7 Load Flow Solution for 14 bus system

Bus Voltage (pu)	**Phase Angle (deg.)**
1.060000	0.000000
1.045000	-4.987322
1.007684	-12.728554
1.014397	-10.291212
1.018885	-8.756863
1.045207	-14.300227
1.056075	-13.481197
1.090000	-13.481197
1.046096	-15.144455
1.038435	-15.297684
1.038315	-14.946079
1.031168	-15.191271
1.027371	-15.296307
1.019671	-16.244354

Table 3.8 Comparison of different methods for 14 bus system

METHOD	OTHER VARIABLES	ITERATIONS
FIRST ORDER	--	4
METHOD 1	--	4
METHOD 3	--	4
METHOD 4	α=1.0 β=0.95	4
METHOD 5(a)	--	4
METHOD 5(b)	--	4
METHOD 5(c)	--	3
METHOD 6	--	3
DECELERATED CONVERGENCE METHOD	α=1	4
TWO-STEP ALGORITHM	α=0.6, C=0.001	3
THREE-STEP ALGORITHM	α=0.6, C=0.0001	2
CONSTANT POSITIVE SHIFT ALGORITHM	S=0.0005	4
INCREASING POSITIVE SHIFT ALGORITHM	S=0.0005,C=0.2	4

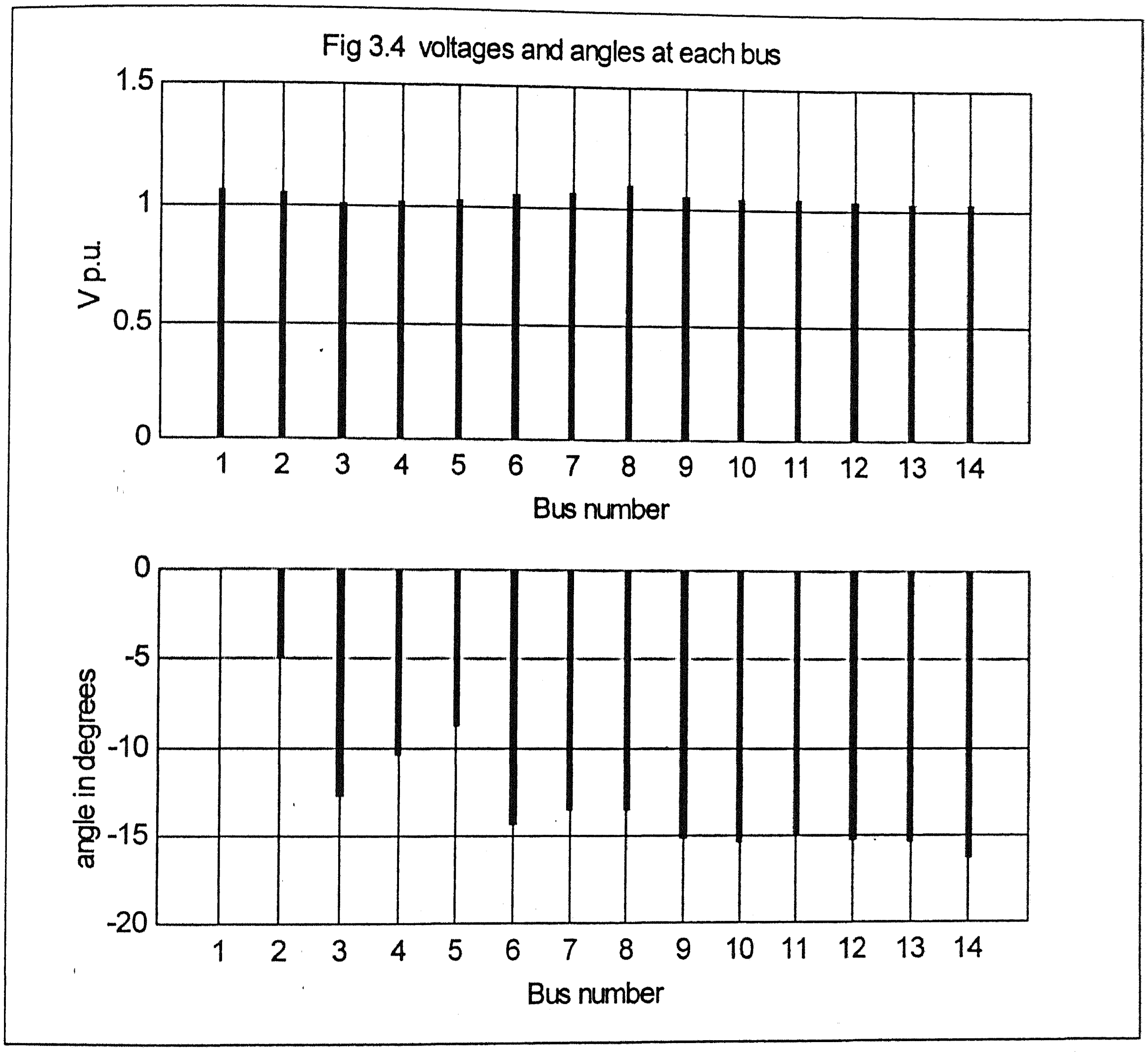


Fig 3.4 voltages and angles at each bus
1.5
1
0.5
0
V p.u.
1 2 3 4 5 6 7 8 9 10 11 12 13 14
Bus number
0
-5
-10
-15
-20
angle in degrees
1 2 3 4 5 6 7 8 9 10 11 12 13 14
Bus number

CHAPTER 4

GENERATION PRICING IN SINGLE AREA CONTROL

4.1 INTRODUCTION

Power being a complex quantity, it has been observed that any change in real power output of the generators directly affect the system frequency and tie-line loadings while any change in reactive power affect only the system voltage. This property helps in dividing the control problem of a power system into two separate channels: the MW-frequency control channel and the MVAR-voltage control channel. The control of the real power output of the generators in response to change in system frequency and the tie-line power flow so as to maintain the scheduled system frequency and the tie-line interchange with in permitted limit is termed as Automatic Generation Control (AGC) or Load Frequency Control (LFC) [4]. On the other hand the control of reactive power generation in the system in order to maintain constant voltage is known as excitation Control.

The main objective of the AGC for an inter connected power system can be stated as follows

1. Matching generation to load
2. Regulating system frequency error to zero
3. Distributing generation amongst areas so that inter-area tie-line flows match a prescribed schedule.
4. Distributing generation with in each area such that the operating cost of area is minimized.

Automatic Generation Control therefore can be subdivided into two separate control problems. One is, the traditional Load Frequency Control problem, which meets the first three objectives. Second is, Economic Load dispatch problem, which takes care of the last objective.

Generally objective (1) is achieved by the system governor. However, system governor alone is found inadequate to take care of objectives (2) and (3). Therefore, supplementary control is added to the system governor utilizing PID controller such that

the deviations in frequency and tie-line loading from the prescheduled values, following a sudden load change in any area, are reduced to zero in the time span of less than a minute. The controller design should be such that not only the objectives (2) and (3) should be met but also the transient oscillations in frequency are kept to a minimum.

In a competitive market there will be independent generators in a large number. Every producer will take his own decisions while supplying power to the corresponding consumers. He will fix the cost per unit generation depending upon the conditions in which the generators are being operated and the changes in the demand. In every aspect he will try to increase his profit and at the same time he will keep in mind the competition from the other producers. Industry restructuring, and particularly the deregulation of generation, is opening the power sector to market forces.

To coordinate distributed generator actions in the short-term operations and control of the spot energy and ancillary services markets, a price model is proposed. The method used is developed to accurately capture the cost associated with local deviations from the scheduled power and energy. A mathematical framework for price model, designed to coordinate distributed generators as they participate in both the short run energy market and the ancillary services market. This mathematical framework will take care of both frequency change as well as load change.

4.2 LOAD FREQUENCY CONTROL

4.2.1 MODEL OF GOVERNOR

The speed governing system can be modeled as in Fig 4.1

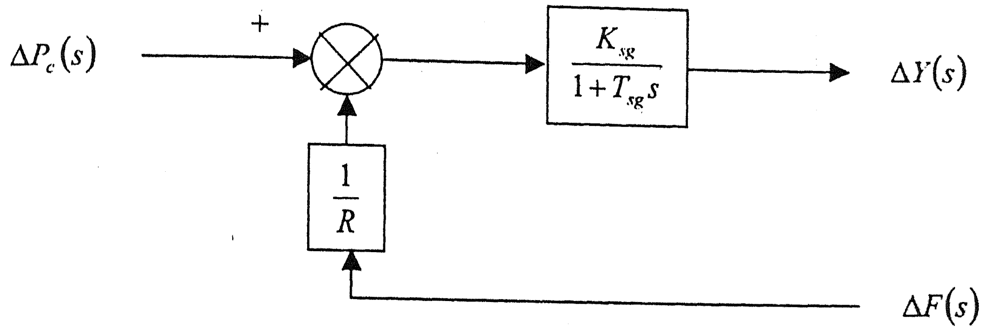


Fig. 4.1 Model of Speed Governor

Where

R=speed regulation of the governor

K_{sg} =gain of speed governor

T_{sg} =time constant of speed governor

Δp_C is the commanded increase in power. This signal sets into motion a sequence of events. When the load increases the frequency goes down. As a result turbine generator speed decreases. Then this signal comes into picture to increase the steam input thus increasing speed, which in turn increases, the frequency.

ΔY is the output signal of the governor. i.e., input signal to the turbine.

4.2.2 MODEL OF TURBINE

The model requires a relation between changes in power output of the steam turbine to changes in its steam valve opening. A non-reheat turbine with a single gain factor K_t and a single time constant T_t is considered. Thus the transfer function of the turbine is

$$G_t(s) = \frac{K_t}{1+T_t s} \quad \text{.........(4.1)}$$

The block diagram representation is shown in Fig 4.2

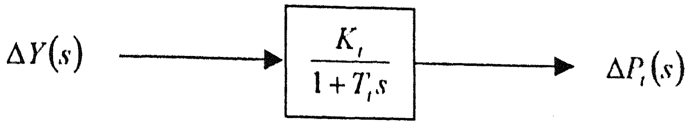


Fig. 4.2 Model of Turbine

4.2.3 GENERATOR LOAD MODEL

The increment in power input to the generator-load system is

$$\Delta P_G - \Delta P_D$$

where $\Delta P_G = \Delta P_t$, incremental turbine power output and ΔP_D is the load increment.

As the frequency changes, the motor load changes being sensitive to speed, the rate of change of load with respect to frequency, i.e., $\partial P_D / \partial f$ can be regarded as nearly constant for small changes in frequency Δf and can be expressed as

$$(\partial P_D / \partial f)\Delta f = \mathbf{B}\,\Delta f$$

where the constant B can be determined empirically. B is positive for a motor load.

Writing the power balance equation,

$$\Delta P_G - \Delta P_D = \frac{2\,H\,P_r}{f^0}\frac{d}{dt}(\Delta f) + B\,\Delta f \quad \text{.......(4.2)}$$

$$\Delta P_G(pu) - \Delta P_D(pu) = \frac{2H}{f^0}\frac{d}{dt}(\Delta f) + B(pu)\Delta f \quad \text{......(4.3)}$$

Taking Laplace transform,

$$\Delta F(s) = \frac{\Delta P_G(s) - \Delta P_D(s)}{B + \frac{2H}{f^0}s} \quad \text{........(4.4)}$$

$$\Delta F(s) = [\Delta P_G(s) - \Delta P_D(s)]\left(\frac{K_{ps}}{1+T_{ps}\,s}\right) \quad \text{......(4.5)}$$

where

$$T_{ps} = \frac{2H}{Bf^0} = \text{Power system time constant}$$

$$K_{ps} = \frac{1}{B} = \text{power system gain}$$

The block diagram representation is shown in Fig 4.3

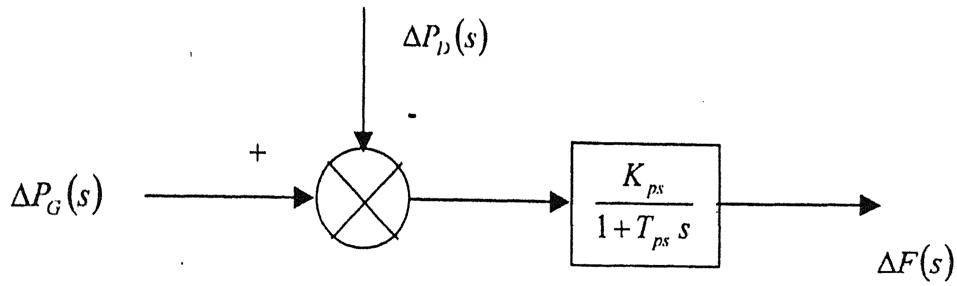


Fig. 4.3 Model of Generator-Load

4.2.4 COMPLETE BLOCK DIAGRAM OF LFC FOR SINGLE AREA

A complete block diagram representation of an isolated power system comprising turbine, generator, governor and load is easily obtained by combining the block diagrams of individual components. The complete block diagram with feedback loop is shown in fig (4.4).

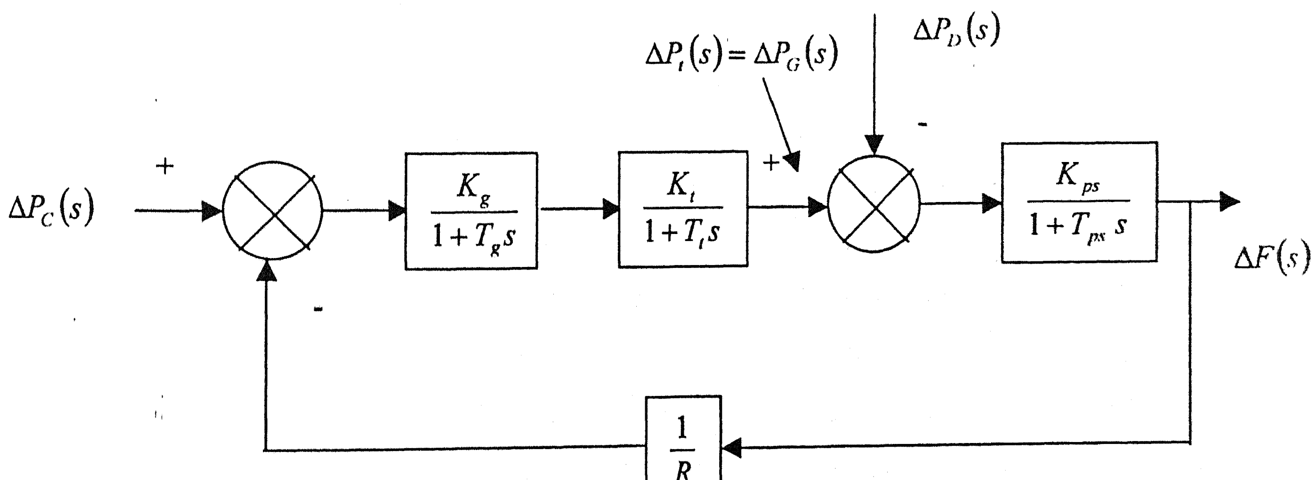


Fig. 4.4 Block diagram representation of Load Frequency Control

4.2.5 STEADY STATE ANALYSIS

Two important incremental inputs to the load frequency control system are - ΔP_C, the change in speed changer setting; and ΔP_D, the change in load demand. If a simple situation in which the speed changer has a fixed setting (i.e., ΔP_C=0) and the load demand changes. This is known as free governor operation. For such an operation the steady state change in system frequency for a sudden load change by an amount ΔP_D is

$$\Delta F(s) = -\frac{K_{ps}}{\left(1+T_{ps}s\right)+\dfrac{K_g K_t K_{ps}/R}{\left(1+T_g s\right)\left(1+T_t s\right)}} \times \frac{\Delta P_D}{s} \qquad \text{at } \Delta P_C(s)=0$$

..................(4.6)

$$\Delta f = -\left(\frac{K_{ps}}{1+\left(K_g K_t K_{ps}/R\right)}\right)\Delta P_D$$ is steady state frequency

..............(4.7)

4.2.6 DYNAMIC RESPONSE

To obtain the dynamic response giving the change in frequency as function of the time for a step change in load, the Laplace inverse of Eq. (4.6) is to be calculated.

4.2.6.1 PID CONTROLLER

The droop in the frequency from no load to full load should be as small as possible as much change in frequency cannot be tolerated. In fact the steady change in the frequency should be zero. While steady state frequency can be brought back to the scheduled value by adjusting speed changer setting, the system could undergo intolerable dynamic frequency changes with changes in load. It leads to the natural suggestion that the speed changer setting be adjusted automatically by monitoring the frequency changes. For this purpose, a signal from Δf is fed through a PID controller to the speed changer. The above modification makes the steady state error to fall to zero and settling time to reduce.

Therefore,

$$\Delta P_C(s) = \left(-K_1 - K_2/s - K_3 s\right) b\, \Delta F(s) \quad \text{........(4.8)}$$

where b is feed back gain.

K_1 is gain of proportional controller

K_2 is gain of integral controller

K_3 is gain of differential controller

In this case the change in frequency will be fed back to the PID controller the output of the output of the controller is $\Delta P_C(s)$.

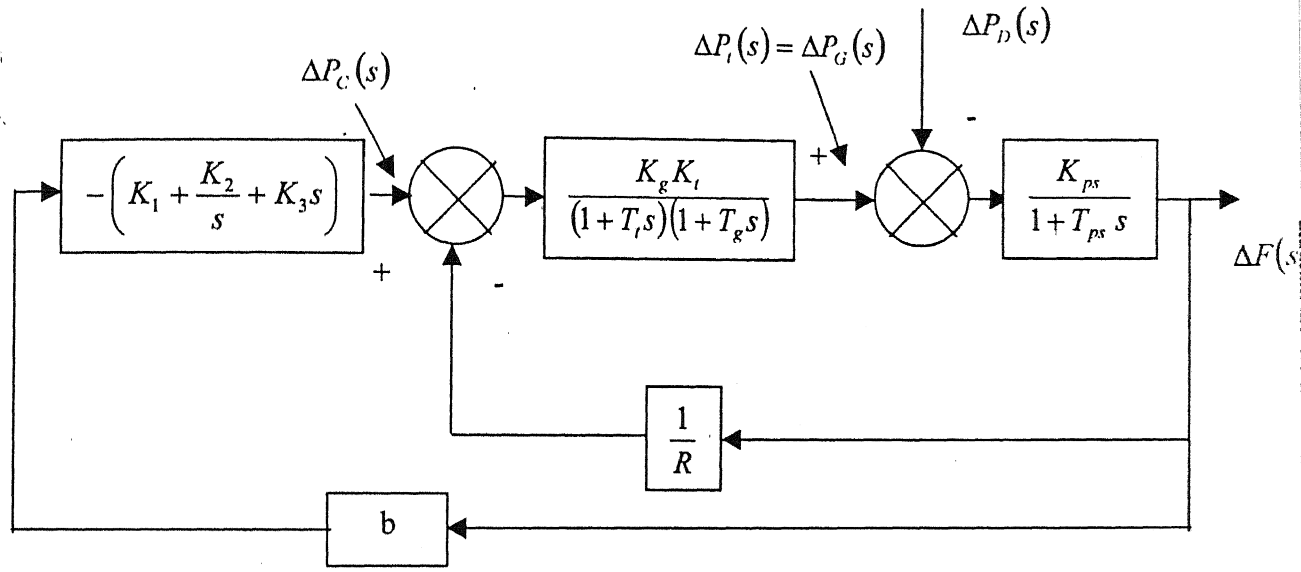


Fig. 4.5 Load Frequency Control with PID controller

The signal $\Delta P_C(s)$ generated by the PID controller must be opposite sign to $\Delta F(s)$ which accounts for negative sign in the block for integral controller. Now

$$\Delta F(s) = -\frac{RK_{ps}s(1+T_g s)(1+T_t s)}{s(1+T_g s)(1+T_t s)(1+T_{ps}s)R + K_{ps}\left(\left(K_1 + \frac{K_2}{s} + K_3 s\right)R + s\right)} \frac{\Delta P_D}{s}$$

$$\Delta f\,|_{\text{steady state}} = \underset{s\to 0}{Lt}\; s\,\Delta F(s) = 0 \quad \text{..... (4.9)}$$

4.3 GENERATION PRICING

With in the distributed system load is distributed throughout the system and generators are located at specific buses. To simulate the dynamic behavior of the system, disturbances are specified as load fluctuations of small magnitude to allow the use of small-signal, liner models. A system model is defined by specifying the distribution system topology, the location and size of loads and the location, size and type of the generators. The inputs to the models are the system disturbances, represented as the input vector to the system of state equations. The output from the simulation is the dynamic behavior of all the state variables, with frequency and real power output typically being of greater interest than the others. Monitoring the frequency at each bus can assess the frequency stability.

The goals in developing models for analyzing frequency behavior are to represent the dynamics of distributed generators in response to system disturbances, and to propose and analyze the effectiveness of different control strategies designed to ensure system stability.

The modeling effort is based on building decoupled, linearized state space models for each distributed generator and coupling them through a distribution system model. The models that include a synchronous generator all use a form of the swing equation as the generator state equation:

$$J\ddot{\delta} + D\dot{\delta} = P_m - P_e \qquad \text{..........(4.10)}$$

Where $P_e \equiv P_G$, the electrical power output.

P_m is the mechanical power from the turbine.

The turbines in the system are assumed as steam turbines to continue the following analysis. In developing the model of the generator, the objective is to represent each generator with a small number of state variables.

The small signal dynamic for each generator is

$$
\begin{aligned}
M \dot{w}_G &= (e_t - D)\, w_G + P_t - P_G \\
T_u \dot{P}_t &= -P_t + k_t\, a \qquad \text{..........(4.11)} \\
T_g \dot{a} &= -w_G - r\, a
\end{aligned}
$$

Where

M is the inertia constant

e_t is a coefficient representing the turbine self-regulation defined as $\partial P_t / \partial w_G$

D is the damping coefficient

T_u is the time constant representing the delay between the control valves and the turbine nozzles

k_t is a proportionality factor representing the control valve position relative to the turbine output variation.

T_g is the time constant of the valve-servomotor-turbine gate system

r is the permanent speed droop of the turbine

After getting the deviations in the frequency and generation in each control area following the load changes in every area, the generations cost in every control area are calculated as follows.

The price model presented is for decoupled real power/frequency dynamics. The reason for this is two fold. First, with this emphasis, the modeling effort mirrors the pattern to date for developing a spot price or responsive price system, which usually focuses on pricing real power, since that is the major commodity of the industry. The second reason is that the use of distributed generators for voltage support is reasonably well accepted by the power industry. In contrast, the frequency dynamics of distribution systems with distributed generation units, and the possibility of these units participating in the supply of ancillary services such as frequency stability and spinning reserve, are relatively new issues. Small signal, linearized models are used for analyzing these markets.

The development of the price model begins here by expressing the cost of power generation in terms of the state variables in the generator equations. Cost can be incorporated into the state space generator models by writing an output equation to

The development of the price model begins here by expressing the cost of power generation in terms of the state variables in the generator equations. Cost can be incorporated into the state space generator models by writing an output equation to capture the variable costs associated with generating power from any given technology. Each state space model identifies the set of elements that together can reproduce the basic machine performance. The cost equation then will become now,

$$\cos t = C \ = C_w \, w_G + C_p \, P_t + C_a \, a + C_g \, P_G \qquad \text{..........(4.12)}$$

The coefficients in the above equation represent the marginal cost associated with each piece of equipment or process represented by the specified state variable.

C_g is the marginal fuel cost

C_w, C_p and C_a are the marginal cost coefficients associated with state variables W_G, P_t (mechanical power output of the turbine) and a (governor valve position) respectively.

With the addition of the output cost equation, the model for each generator can be expressed as

$$M \dot{w}_G = (e_t - D)\, w_G + P_t - P_G$$

$$T_u \dot{P}_t = -P_t + k_t \, a$$

$$T_g \dot{a} = -w_G - r\,a + w_{ref} \qquad \text{..........(4.13)}$$

$$\cos t = C \ = C_w \, W_G + C_p \, P_t + C_a \, a + C_g \, P_G$$

The above model can be represented as

$$\begin{aligned} \dot{x} &= f(x\,,\, P_G) \\ C &= h(x\,,\, P_G) \end{aligned} \qquad \text{..........(4.14)}$$

The generators and the system will respond to the price signal at specific intervals, indicating that the price signal is best modeled in discrete time. The first step in developing this discrete time model is to assume the primary dynamics have settled. Therefore the Eq. (4.14) will now become

$$\begin{aligned} 0 &= f(x\,,\, P_G) \\ C &= h(x\,,\, P_G) \end{aligned} \qquad \text{..........(4.15)}$$

Solving these equations for cost results in a discrete time cost equation of the form

$$C(k) = \gamma_1 \, w_G(k) + \gamma_2 \, P_G(k) + \gamma_3 \, w_{ref} \qquad \text{..........(4.16)}$$

$$\gamma_1 = \left(C_w - \frac{C_p k_t}{r} - \frac{C_a}{r} \right)$$

$$\gamma_2 = C_g \qquad \text{..........(4.17)}$$

$$\gamma_3 = \left(\frac{C_p k_t}{r} - \frac{C_a}{r} \right)$$

For forming the dynamic model, by writing equation (4.16) for two sequential time steps and subtracting. The dynamic equation for the price of energy supplies at a generator is expressed as

$$C(k+1) = C(k) + \gamma_1 \left[w_{Gi}(k+1) - w_{Gi}(K) \right] + \gamma_2 \left[P_{Gi}(k+1) - P_{Gi}(k) \right] \qquad \text{..........(4.18)}$$

4.4 RESULTS

Change in load at any bus will ultimately increase load on generators, this change will cause frequency change also. Current price model has been used to calculate the generation cost considering both the changes. A 31 bus system has been taken for this study. Data for this system is shown in appendix. For a step change of 0.05 p.u. in load at some bus load flow has been run to find out the actual load change that will occur at generator, this change has been taken as $\Delta P_D(s)$ and frequency deviation and generation change have been calculated using the method explained in section 4.2. Using these quantities generation pricing has been done as explained in section 4.3. Results are shown below. Fig 4.6 gives the frequency deviation after step load change, Fig 4.7 gives the change in generation, Fig 4.8 gives the actual frequency, Fig 4.9 gives actual generation and Fig 4.10 gives the price. For all these plots X-axis will be time in seconds. Fig A.4 shows the 31 bus system. Table A.11 gives bus data and Table A.12 gives line data.

By using Controller this frequency deviation can be brought back to zero, here they have not been used. So these coefficients have been taken as 0. Due to this frequency settled to a value less than base value. The values for frequency, generation and price are tabulated below before disturbance, after disturbance and their peak values in table 4.1

Table 4.1 Comparison table

Quantity	Before disturbance	After disturbance (Steady state)	Peak value
Frequency (Hz)	50	49.8231	49.7117
Generation (p.u.)	16.393	16.452	16.4752
Price (Rs.)	22564	22631	22658

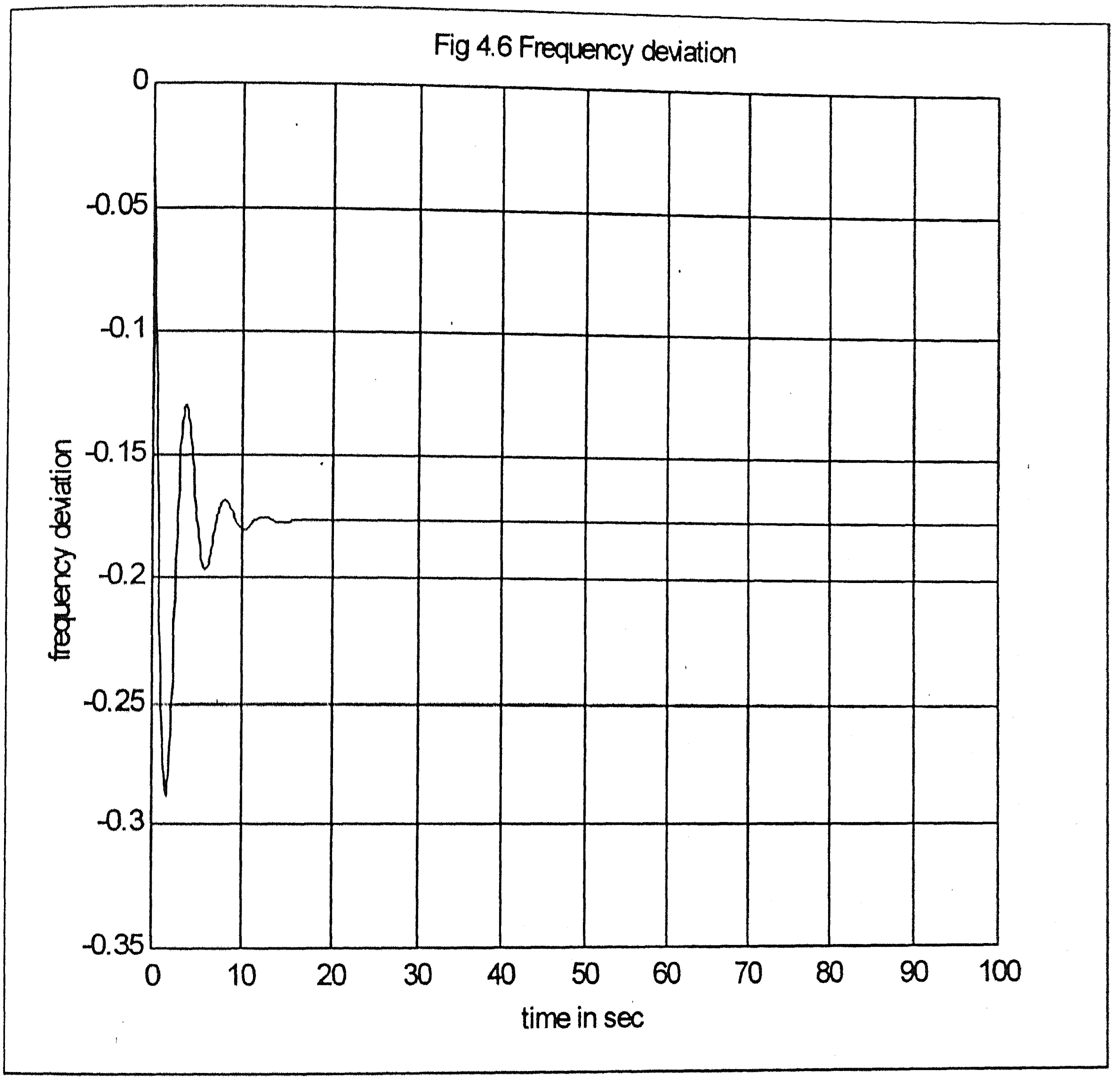


Fig 4.6 Frequency deviation
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
-0.35
frequency deviation
0
10
20
30
40
50
60
70
80
90
100
time in sec

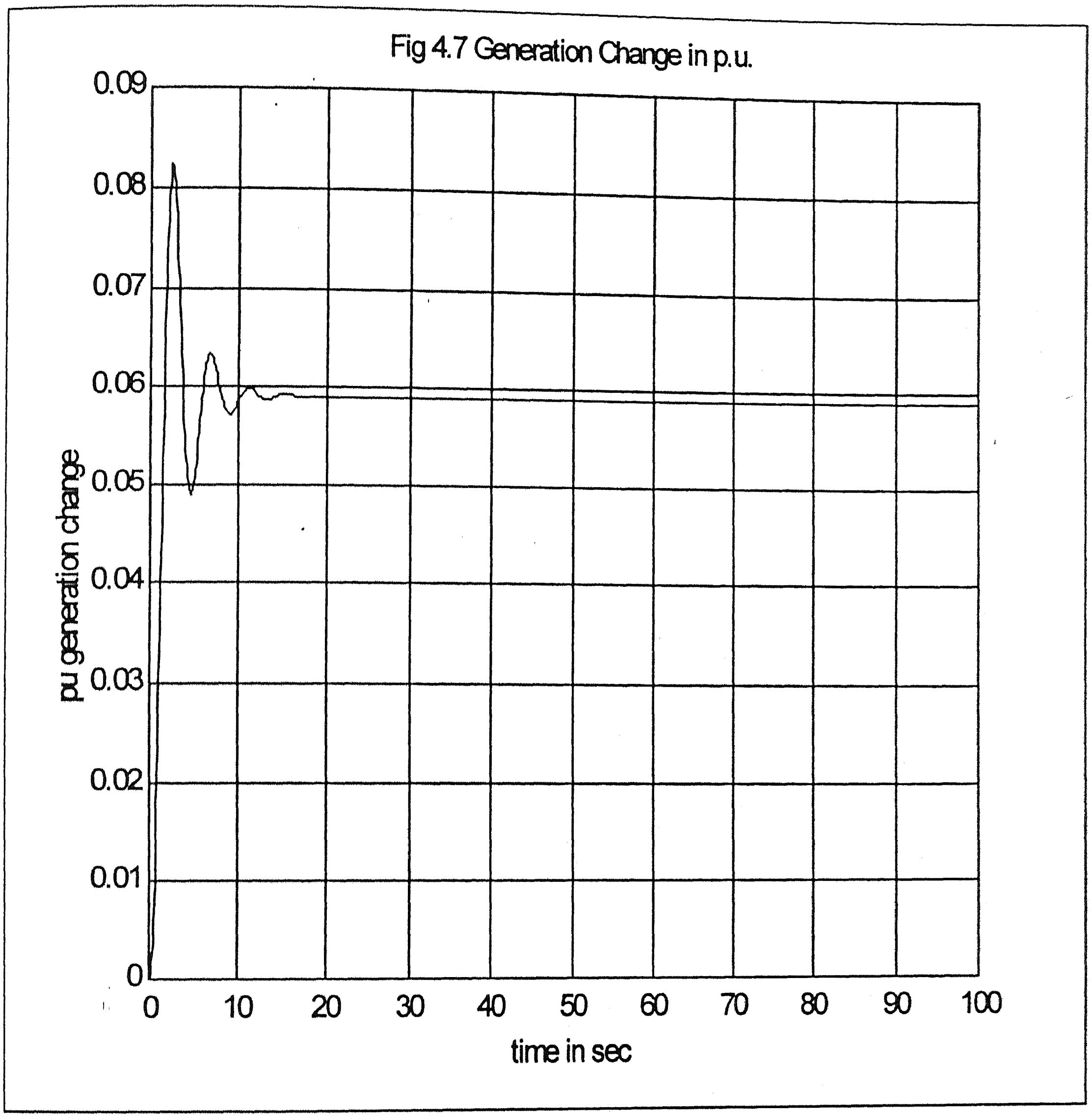


Fig 4.7 Generation Change in p.u.
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
pu generation change
0
10
20
30
40
50
60
70
80
90
100
time in sec

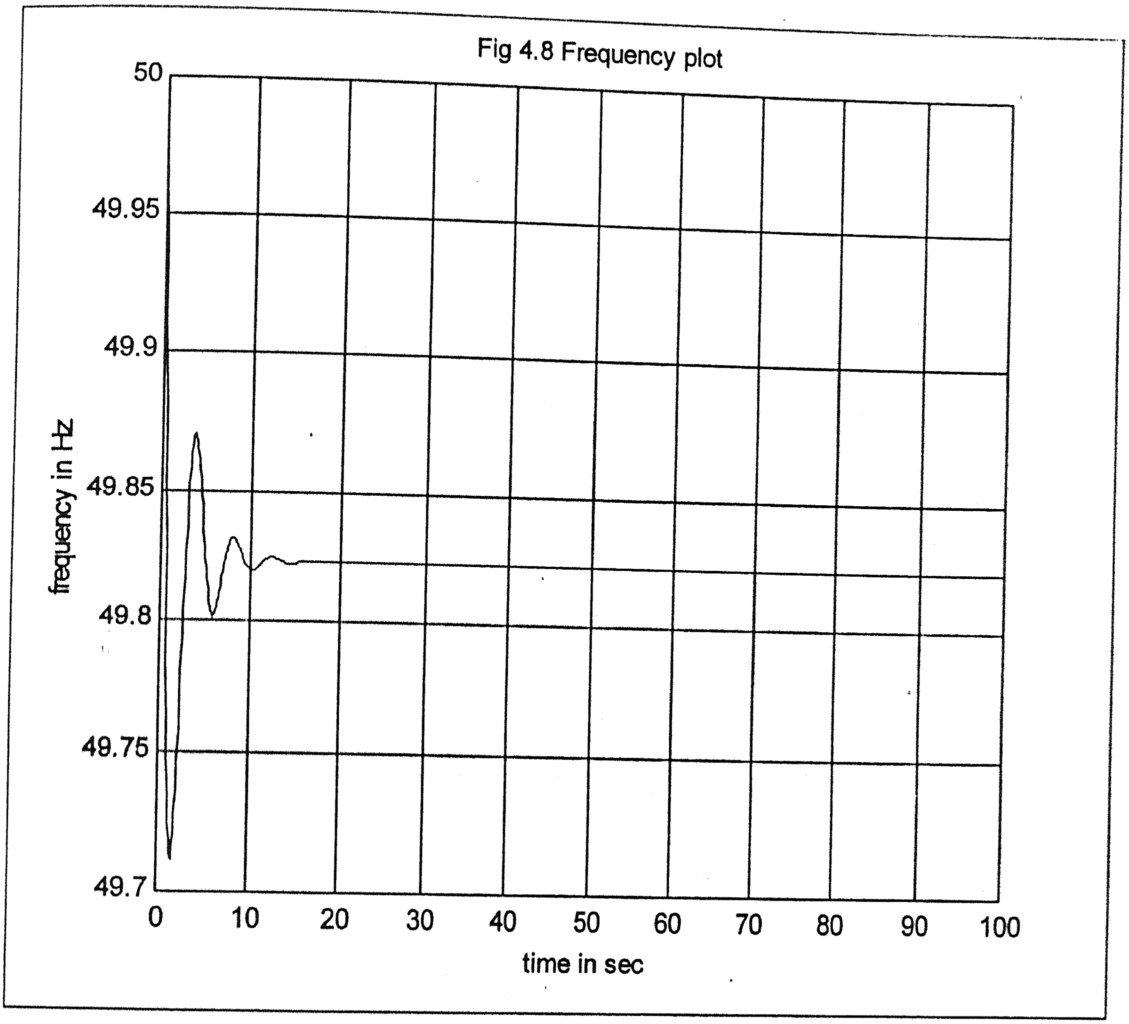


Fig 4.8 Frequency plot
50
49.95
49.9
49.85
49.8
49.75
49.7
frequency in Hz
0
10
20
30
40
50
60
70
80
90
100
time in sec

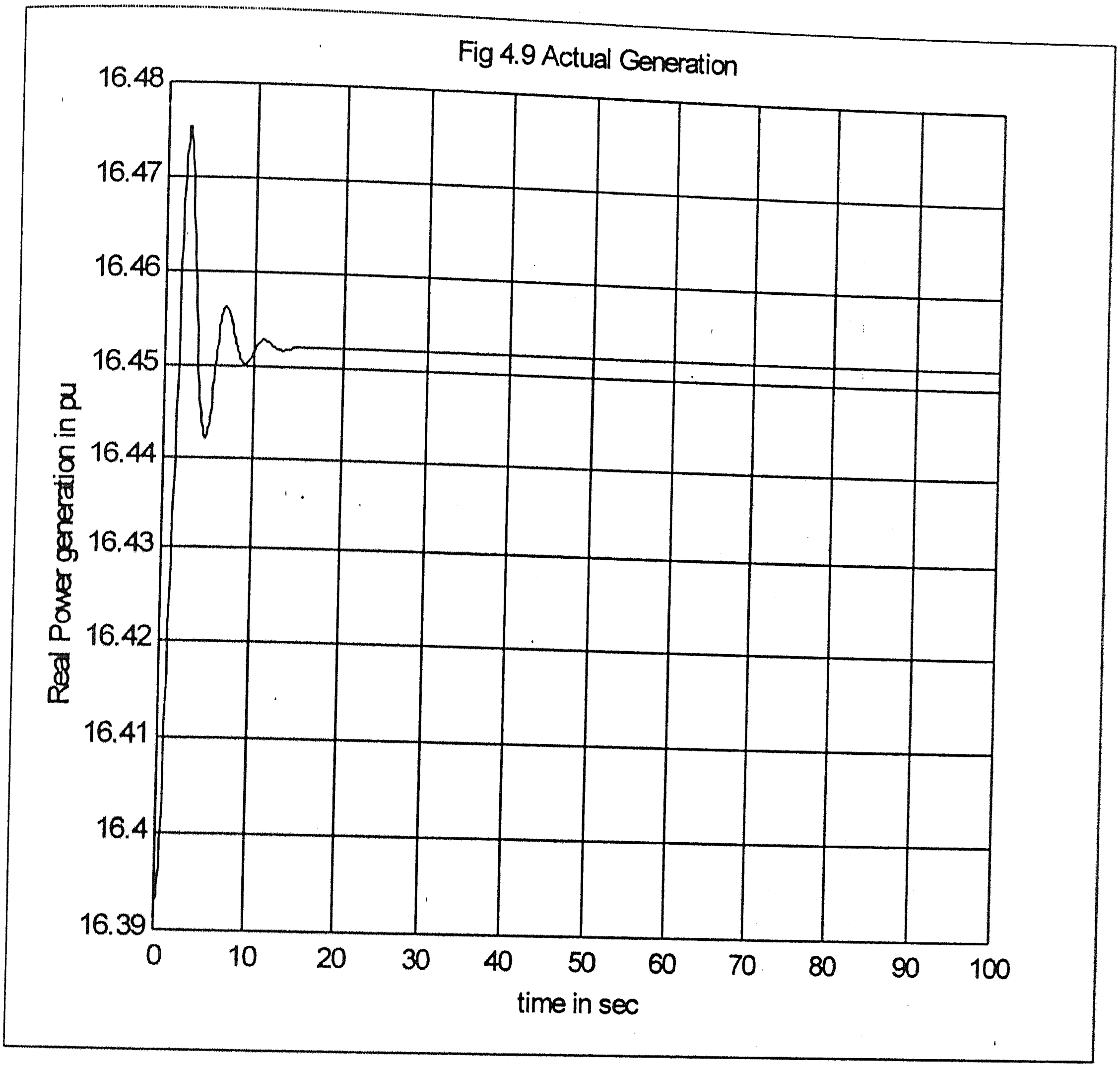
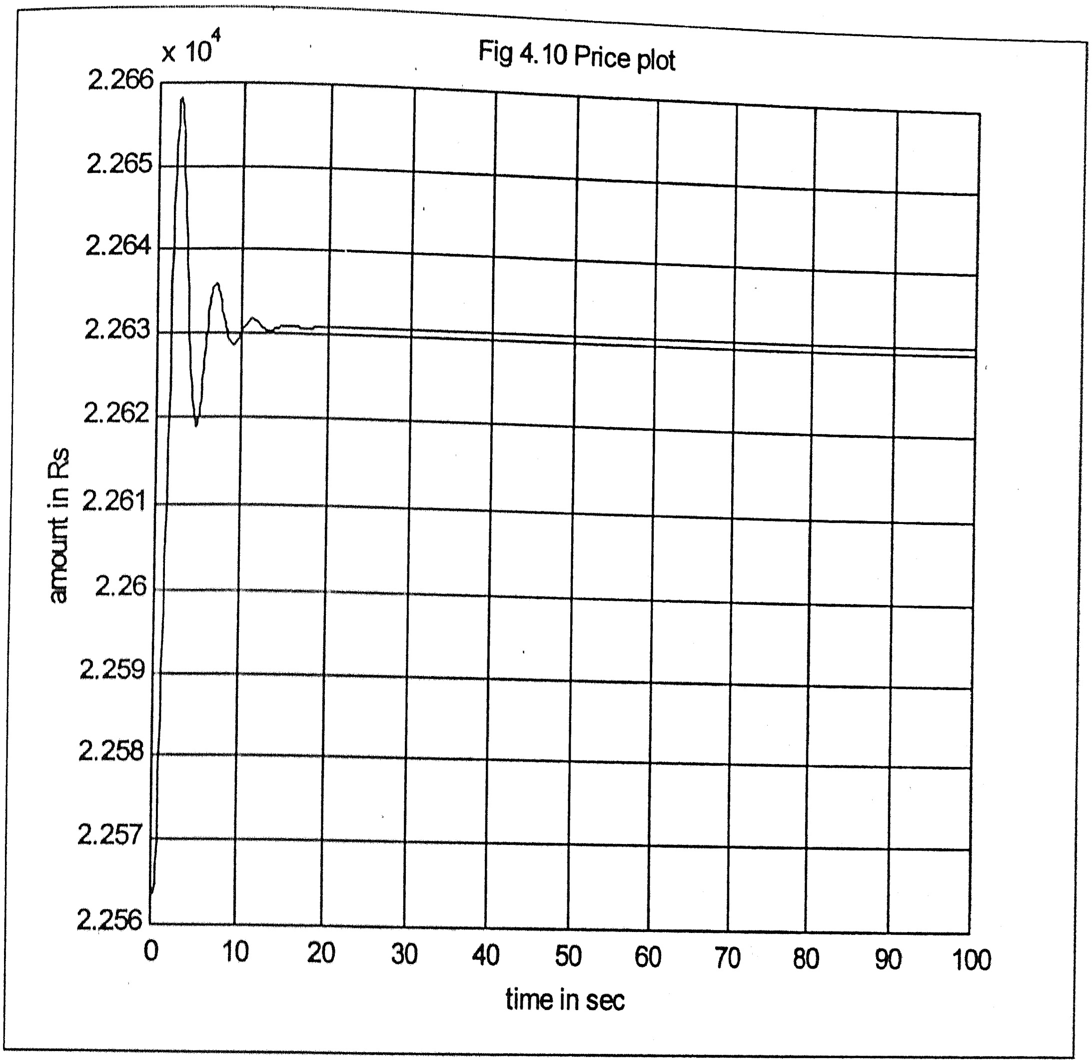


Fig 4.9 Actual Generation
16.48
16.47
16.46
16.45
16.44
16.43
16.42
16.41
16.4
16.39
Real Power generation in pu
0
10
20
30
40
50
60
70
80
90
100
time in sec


x 10^4
Fig 4.10 Price plot
2.266
2.265
2.264
2.263
2.262
2.261
2.26
2.259
2.258
2.257
2.256
amount in Rs
0
10
20
30
40
50
60
70
80
90
100
time in sec

CHAPTER 5

CONCLUSIONS

The main objective of this thesis work is to improve the first-order Newton-Raphson method, to apply it to the power system load flow problem and to do generation pricing considering both frequency deviation and generation change.

Chapter 2 explains modified Newton-Raphson techniques the results shown in this chapter clearly show that the proposed methods work better than the existing one in most of the cases. Proposed methods converged when the first order methods failed to converge and they have taken less number of iterations also. Only when the initial guess is very near to the solution point first-order method and most of the proposed methods took equal number of iterations, but when the initial guess is far away from the final solution, and in case when jacobian becomes singular most of the proposed methods shown improvement over the first-order method this is because of consideration of second order terms through Hessian matrix.

Chapter 3 explains how to apply these methods to power system load flow problem. Proposed methods have shown considerable improvement over the existing first order Newton-Raphson method. First order method failed to converge in case when R/X ratio is greater than 1 whereas some of the proposed methods converged. Most of the proposed methods have taken less number of iteration compared to First-order method. From the results shown in chapter 3, in case of 13-bus system when the resistance value increased to make R/X value greater than 1 First-order method failed to converge and most of the proposed methods converged. For 11-bus and 14-bus system some proposed methods took less number of iteration for base case also.

Chapter 4 explains how to do generation costing considering frequency variation also. Generation cost calculated for a 31 bus test system. From the graphs shown it is clear that when the frequency decreases the cost will increase, when generation increases cost will increase and vice-versa. Therefore the generation cost was made a function of both frequency deviation and real power generation.

For validating the proposed methods they are tested on 43 bus and 31 bus systems and by changing R/X ratio their performance was checked. Results are shown in Appendix B.

SCOPE FOR FURTHER WORK

- Validation for convergence was not explained in the present work, one can do this to show the proposed methods will converge always.
- For the proposed method no mathematical explanation was given to prove how the proposed methods going to be better, validation has done only with respect to result. So one can give mathematical proof.
- Pricing was done only for single control area problem this can be extended to multi area control problem and thus it can be successfully applied in deregulated market.

Appendix A

Data For 13 Bus Test System

The 13 bus system is shown in Fig. A.1. The system data is taken from ref. [1,2]. The relevant data are provided in following tables. Table A.1 gives bus data, Table A.2 gives Line data and Table A.3 gives Transformer data.

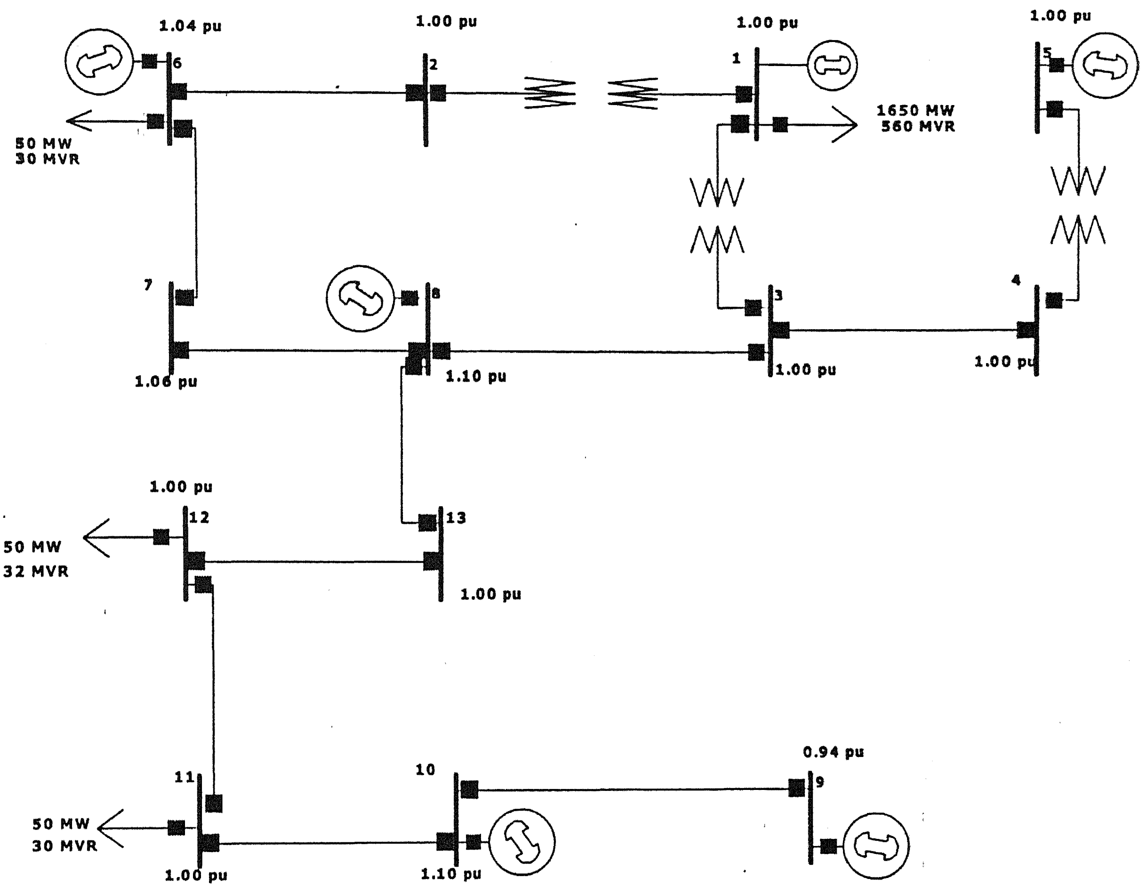


Fig. A.1. 13 Bus power system

Table A.1. Bus data for 13 bus system

Bus No.	Bus Voltage (pu)	Phase Angle (deg.)	Generation		Load	
			Real Power (pu)	Reactive Power (pu)	Real (pu)	Reactive (pu)
1	1.0*	0.0	-	-	1.65	0.56
2	1.0	0.0	0	0	0	0
3	1.0	0.0	0	0	0	0
4	1.0	0.0	0	0	0	0
5	1.0	0.0	0	0	0	0
6	1.037*	0.0	0.5	-	0.05	0.03
7	1.063	0.0	0	0	0	0
8	1.1*	0.0	0	-	0	0
9	0.943*	0.0	0.5	-	0	0
10	1.1*	0.0	0	-	0	0
11	1.0	0.0	0	0	0.05	0.05
12	1.0	0.0	0	0	0.05	0.032
13	1.0	0.0	0	0	0	0

*Input data

Table A.2. Line data for 13 bus system

Branch Number	From Node	To Node	Resistance (pu)	Reactance (pu)	Susceptance (pu)
1	1	2	0.0040	0.085	0
2	1	3	0.0040	0.0947	0
3	5	4	0.0040	0.0947	0
4	4	3	0.0074	0.1430	0.436
5	6	2	0.0481	0.4590	0.246
6	6	7	0.0090	0.1080	0.016
7	8	3	0.0121	0.233	0.712
8	7	8	0	0.15	0
9	9	10	0.0105	0.2020	0.620
10	10	11	0	-0.15	0
11	11	12	0.0086	0.1665	0.508
12	12	13	0.0075	0.1465	0.448
13	13	8	0	-0.15	0

Base=1000 MVA

Table A.3. Transformer data for 13 bus system

Branch number	From node	To node	Tap setting
1	1	2	+ 5%
2	2	3	+ 10%
3	5	4	+ 10%

DATA FOR 11 BUS SYSTEM

The 11 bus system is shown in Fig. A.2. The system data is taken from ref. [1,2]. The relevant data are provided in following tables. Table A.4 gives bus data and Table A.2 gives Y matrix elements

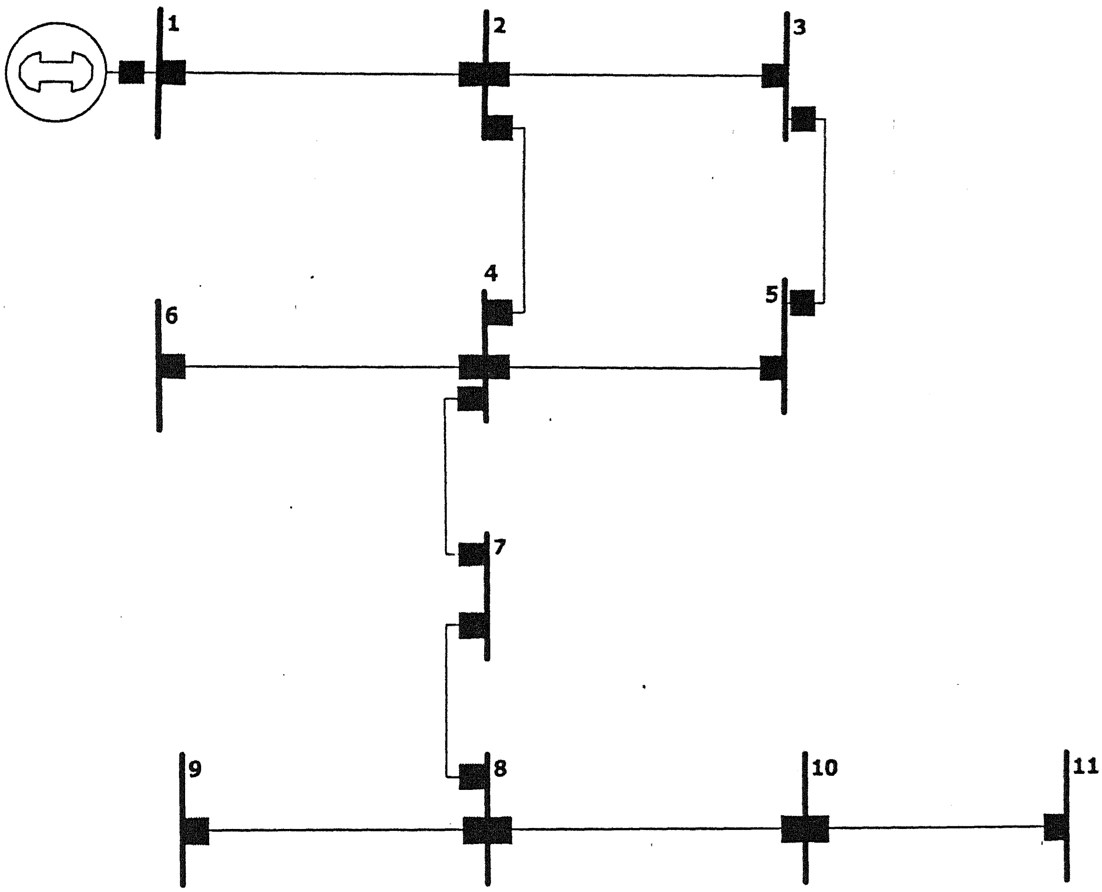


Fig A.2 11 bus system

Table A.4 Bus data for 11 bus system

Bus Number	Voltage (p.u.)	Angle (radians)	Real power injection (p.u.)	Reactive power injection (p.u.)
1	1.024	0	0	0
2	1	0	0	0
3	1	0	-0.128	-0.062
4	1	0	0	0
5	1	0	-0.165	-0.080
6	1	0	-0.090	-0.068
7	1	0	0	0
8	1	0	0	0
9	1	0	-0.026	-0.009
10	1	0	0	0
11	1	0	-0.158	-0.057

Table A.5 Y bus matrix for 11 bus system

From Bus i	To bus j	Conductance G_{ij} (p.u.)	Susceptance B_{ij} (p.u.)
1	1	0.0	-14.939
1	2	0.0	14.148
2	2	12.051	-33.089
2	3	0.0	6.494
2	4	-12.051	13.197
3	3	2.581	-10.282
3	5	-2.581	3.789
4	4	12.642	-74.081
4	5	0.0	2.177
4	6	0.0	56.689
4	7	-0.592	0.786
5	5	2.581	-5.889
6	6	0.0	-55.556
7	7	3.226	-4.304
7	8	-2.213	2.959
8	8	2.893	-5.468
8	9	-.138	1.379
8	10	-0.851	1.163
9	9	0.104	-1.042
10	10	1.346	-6.11
10	11	-0.374	3.742
11	11	0.283	-2.785

DATA FOR 14 BUS SYSTEM

The 14 bus system is shown in Fig A.3, data is taken from ref. [9] and buses are renumbered. The relevant data are provided in following tables. Table A.6 gives Generator Bus Voltages, Table A.7 gives Generator Data, Table A.8 gives Transformer Data, Table A.9 gives Load Bus data and Table A.10 gives Line Data.

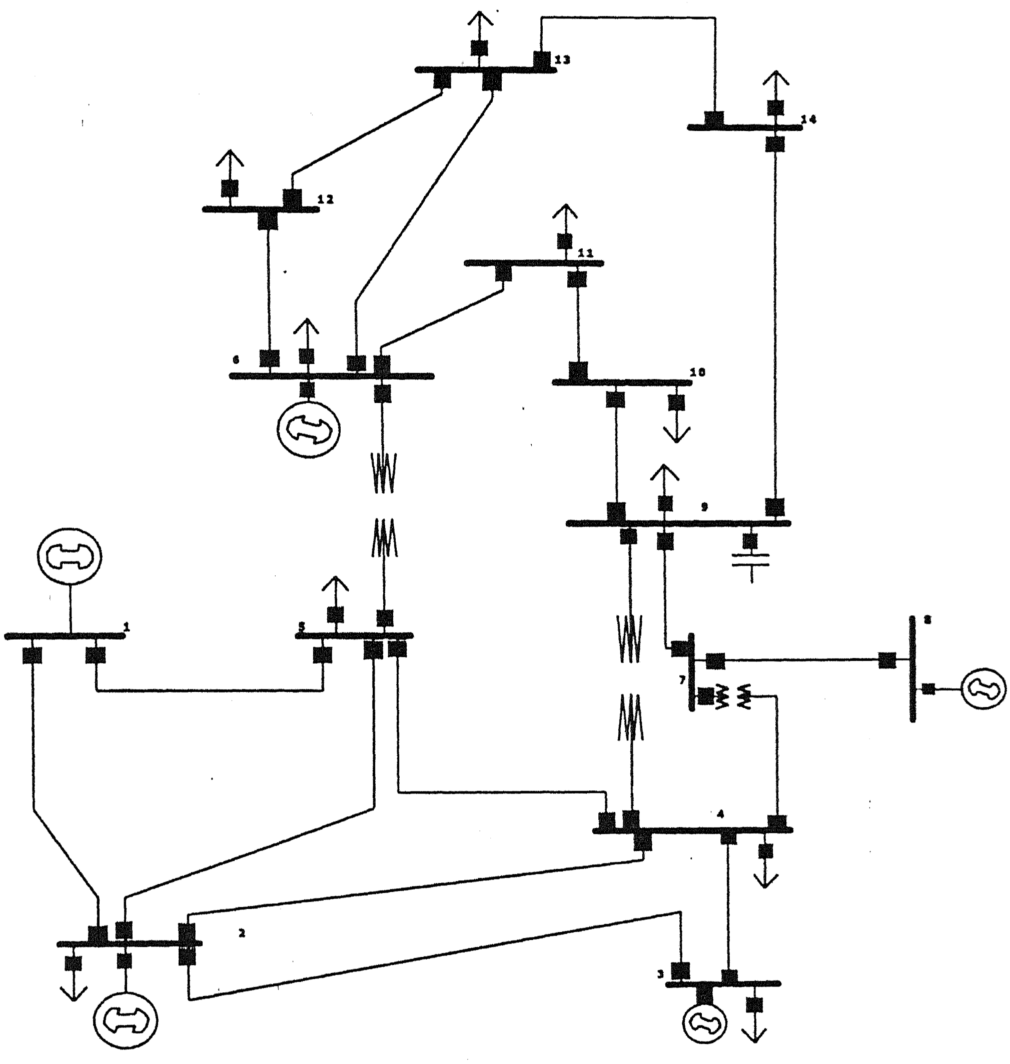


Fig A.3 14 bus system

Table A.6: Generator Data for 14 bus system

Generator No	Real Power Generation Limit		Reactive Power Generation Limit	
	Maximum (MW)	Minimum (MW)	Maximum (MW)	Minimum (MW)
1	200.00	050.0	100.0	-45.0
2	100.0	020.0	050..0	-40.0
3	-	-	040.0	00.0
6	150.0	020.0	024.0	-06.0
8	-	-	024.0	-06.0

Table A.7: Generator Bus Voltages for 14 bus system

Bus No.	Scheduled Real Power Generation P_G(MW)	Specifieed Voltage Magnitude V_{spec}(p.u)	Load	
			Real (MW)	Reactive (MVAR)
1	-	1.060	00.00	00.00
2	40.0	1.045	21.70	12.70
3	-	1.010	94.20	19.00
6	20.0	1.070	11.2	7.5
8	-	1.090	00.00	00.00

Table A.8: Transformer Data for 14 bus system

Line No	From Bus	To Bus	Series Impedence		Tap Setting
			Resistance (0.u)	Reactance (p.u)	
8	4	7	0.0	0.2091	0.978
9	4	9	0.0	0.5561	0.969
10	5	6	0.0	0.2502	0.962

Table A.9 Load Bus Data for 14 bus system

Bus no	Load		External Shunt Susceptance (p.u)
	Real	Reactive	
4	47.8	4.0	0.0
5	7.6	1.6	0.0
7	0.0	0.0	0.0
9	29.5	16.6	0.19
10	9.0	5.8	0.0
11	3.5	1.8	0.0
12	6.1	1.6	0.0
13	13.5	5.8	0.0
14	14.9	5.0	0.0

Table A.10: Line Data for 14 bus system

Line. No.	From No	To Bus	Series Impedance		Shunt Susceptance (p.u)
			Resistance (p.u)	Reactance (p.u)	
1	1	2	0.01938	0.05917	0.528
2	1	5	0.05403	0.22304	0.0492
3	2	3	0.04699	0.19797	0.0438
4	4	4	0.05811	0.17632	0.0374
5	2	5	0.05695	0.17388	0.0340
6	3	4	0.06701	0.17103	0.0346
7	4	5	0.01335	0.04211	0.0
11	6	11	0.09798	0.19890	0.0
12	6	12	0.12291	0.25581	0.0
13	6	13	0.06615	0.13027	0.0
14	7	8	0.0	0.17615	0.0
15	7	9	0.0	0.11001	0.0
16	9	10	0.03181	0.08450	0.0
17	9	14	0.12711	0.27038	0.0
18	10	11	0.08205	0.19207	0.0
19	12	13	0.22092	0.19988	0.0
20	13	14	0.17093	0.34802	0.0

Data For 31 Bus Test System

The 31 bus system is shown in Fig. A.4. The system data is taken from ref. [5,6]. The relevant data are provided in following tables. Table A.11 gives bus data and Table A.12 gives Line data.

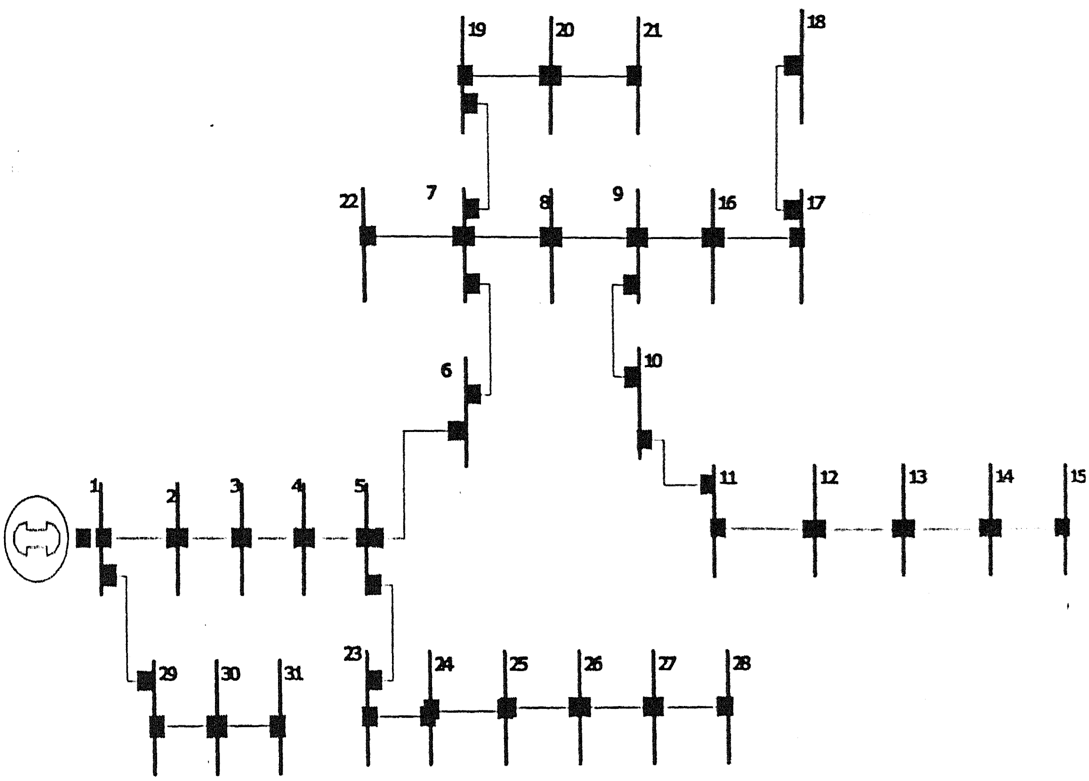


Fig A.4 31 bus radial system

Table A.11 Bus data for 31 bus system

Bus Number	Voltage (p.u.)	Angle (radians)	Real power injection (p.u.)	Reactive power injection (p.u.)
1	1.0	0.0	0	0
2	1.0	0.0	0	0
3	1.0	0.0	-0.522	-0.174
4	1.0	0.0	0	0
5	1.0	0.0	-0.936	-0.312
6	1.0	0.0	0	0
7	1.0	0.0	0	0
8	1.0	0.0	0	0
9	1.0	0.0	0	0
10	1.0	0.0	-0.189	-0.063
11	1.0	0.0	0	0
12	1.0	0.0	-0.336	-0.112
13	1.0	0.0	-0.657	-0.219
14	1.0	0.0	-0.783	-0.261
15	1.0	0.0	-0.729	-0.243
16	1.0	0.0	-0.477	-0.159
17	1.0	0.0	-0.549	-0.183
18	1.0	0.0	-0.477	-0.159
19	1.0	0.0	-0.432	-0.144
20	1.0	0.0	-0.672	-0.224
21	1.0	0.0	-0.495	-0.165
22	1.0	0.0	-0.207	-0.069
23	1.0	0.0	-0.522	-0.174
24	1.0	0.0	-1.917	-0.639
25	1.0	0.0	0	0
26	1.0	0.0	-1.116	-0.372
27	1.0	0.0	-0.549	-0.183
28	1.0	0.0	-0.792	-0.264
29	1.0	0.0	-0.882	-0.294
30	1.0	0.0	-0.882	-0.294
31	1.0	0.0	-0.882	-0.294

Table A.12 Line data for 31 bus system

From Bus i	To Bus j	Series impedance	
		Resistance (p.u.)	Reactance (p.u.)
1	2	0.000963	0.00322
2	3	0.000414	0.00002
3	4	0.000659	0.00065
4	5	0.00222	0.00193
5	6	0.001045	0.000909
6	7	0.003143	0.00177
7	8	0.00255	0.00144
8	9	0.00255	0.00144
9	10	0.002506	0.001412
10	11	0.002506	0.001412
11	12	0.007506	0.00423
12	13	0.003506	0.00198
13	14	0.001429	0.000805
14	15	0.00291	0.001639
9	16	0.000898	0.000781
16	17	0.00138	0.000775
17	18	0.002468	0.00139
7	19	0.000915	0.000795
19	20	0.003	0.00261
20	21	0.00291	0.001639
7	22	0.001143	0.000994
4	23	0.001066	0.001054
23	24	0.000649	0.0006414
24	25	0.00108	0.000941
25	26	0.00276	0.002399
26	27	0.002001	0.001746
27	28	0.002857	0.00161
2	29	0.0008807	0.0000964
29	30	0.003091	0.00174
30	31	0.002106	0.001187

Table A.13 Coefficients used

Parameter	**Value**
M	1.26
D	2.0
k_t	0.95
e_t	0.15
T_u	0.2
t_g	0.25
r	0.05
R	3
C_p	200
C_w	175
C_a	200
C_g	1000
K_{ps}	100
T_{ps}	20
T_g	0.4
T_t	0.5

Data For 43 Bus Test System

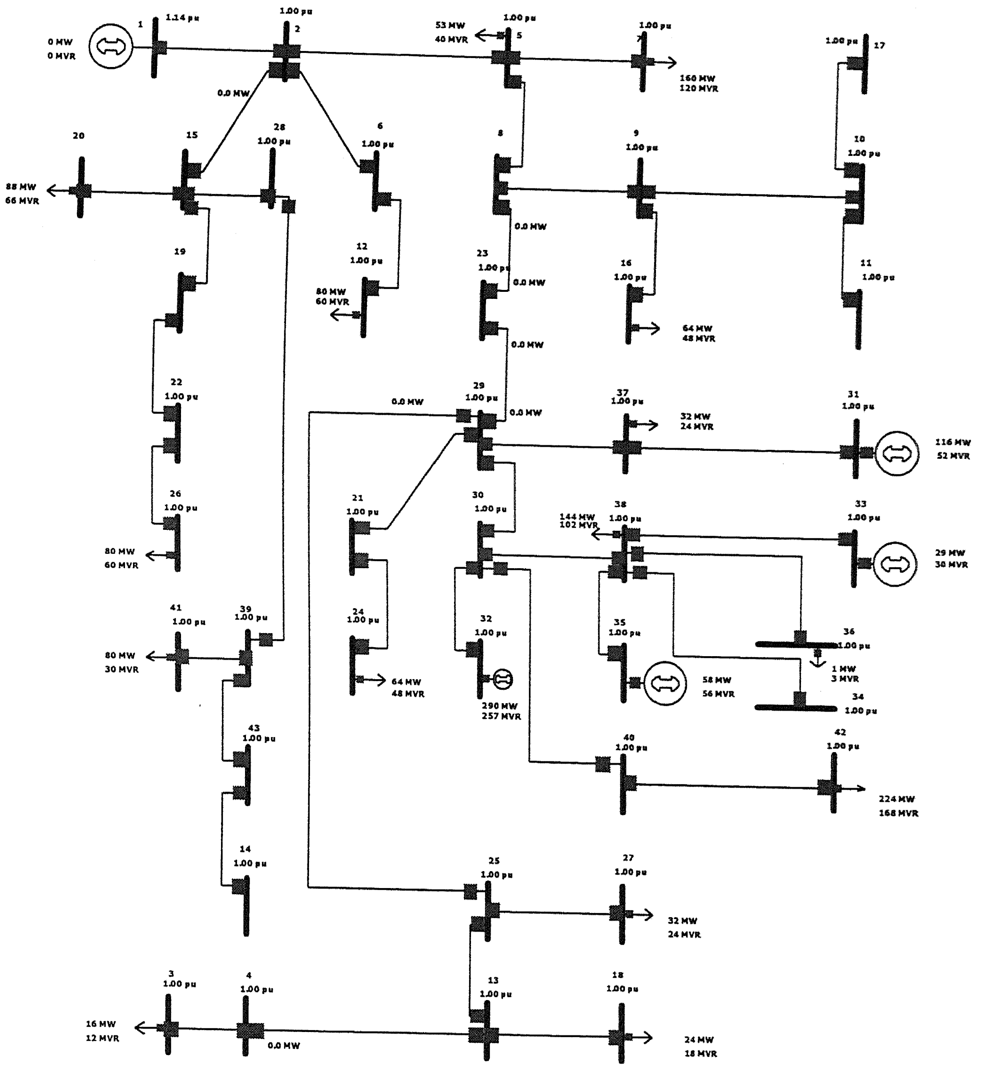


Fig A.5 43 bus test system

The 43 bus system is shown in Fig. A.5. The system data is taken from ref. [4, 7]. The relevant data are provided in following tables. Table A.14 gives Y Bus matrix elements of 43 bus system, Table A.15 gives the operating condition of the system.

Table A.14 Y Bus matrix elements of 43 bus system

From Node (i)	To Node (j)	G_{ij}	B_{ij}
1	1	0	-30.609
1	2	0	30.609
2	2	481.288	-1545.194
2	5	-277.195	873.583
2	6	-34.368	108.124
2	15	-169.726	534.322
3	3	0	-5.714
3	4	0	6.015
4	4	61.331	-69.160
4	13	-61.331	62.874
5	5	277.195	-916.892
5	7	0	21.277
5	8	0	20.513
6	6	34.368	-118.699
6	12	0	10.638
7	7	0	-20
8	8	452.840	-482.861
8	9	-288.938	295.777
8	23	-163.902	167.191
9	9	300.983	-317.044
9	10	-12.045	12.342
9	16	0.0	8.796
10	10	12.045	-20.885
10	11	0.0	2.857
10	17	0.0	5.714
11	11	0.0	-2.857
12	12	0.0	-10.0
13	13	92.381	-100.709
13	18	0.0	6.015
13	25	-31.05	31.640
14	14	0.0	-15.015
14	43	0.0	15.4
15	15	340.398	-916.783
15	19	0.0	8.649
15	20	0.0	15.791
15	28	-170.673	357.003
16	16	0.0	-8.576
17	17	0.0	-5.714
18	18	0	-5.714
19	19	164.292	-280.783

19	22	-164.292	272.805
20	20	0	-15.002
21	21	104.312	-143.609
21	24	0	9.267
21	29	-104.312	133.623
22	22	164.292	-282.281
22	26	0	9.023
23	23	321.579	-328.81
23	29	-157.677	161.76
24	24	0	-8.572
25	25	87.15	-106.814
25	27	0	9.023
25	29	-56.1	65.824
26	26	0	-8.572
27	27	0	-8.572
28	28	373.447	-612.837
28	39	-202.775	256.136
29	29	318.089	-372.311
29	30	0	3.766
29	37	0	7.895
30	30	125.789	-524.464
30	32	0	30.769
30	38	0	4.131
30	40	-125.789	485.547
31	31	0	-13.038
31	37	0	13.038
32	32	0	-30.769
33	33	0	-3.320
33	38	0	3.32
34	34	0	-7.365
34	38	0	6.852
35	35	0	-6.18
35	38	0	6.18
36	36	0	-2.703
36	38	0	2.703
37	37	0	-21.348
38	38	0	-22.398
39	39	512.581	-663.260
39	41	0	15.015
39	43	-309.806	392.255
40	40	125.789	-508.837
40	42	0	21.622
41	41	0	-15.015
42	42	0	-20
43	43	309.806	-408.029

Table A.15 Operating condition of 43 bus system

Bus No.	Voltage magnitude (p.u)	Phase Angle (deg.)	Net Real Power (p.u)	Net Reactive Power (p.u)
1	1.136	0	0	0
2	1	0	0	0
3	1	0	-0.16	-0.12
4	1	0	0	0
5	1	0	-0.53	-0.4
6	1	0	0	0
7	1	0	-1.6	-1.2
8	1	0	0	0
9	1	0	0	0
10	1	0	0	0
11	1	0	0	0
12	1	0	-0.8	-0.6
13	1	0	0	0
14	1	0	-0.8	-0.6
15	1	0	0	0
16	1	0	-0.64	-0.48
17	1	0	0	0
18	1	0	-0.24	-0.18
19	1	0	0	0
20	1	0	-0.88	-0.66
21	1	0	0	0
22	1	0	0	0
23	1	0	0	0
24	1	0	-0.64	-0.48
25	1	0	0	0
26	1	0	-0.8	-0.6
27	1	0	-0.32	-0.24
28	1	0	0	0
29	1	0	0	0
30	1	0	0	0
31	1	0	1.16	0.52
32	1	0	2.9	2.57
33	1	0	0.285	0.3
34	1	0	0	0
35	1	0	0.58	0.56
36	1	0	-0.005	0.030
37	1	0	0	0
38	1	0	-1.44	-1.02
39	1	0	0	0
40	1	0	0	0
41	1	0	-0.8	-0.3
42	1	0	-2.24	-1.68
43	1	0	0	0

Appendix B

Table B.1 Load flow solution for 14 bus system when resistance value increased 4 times

Voltage (p.u.)	Angle (deg.)
1.060000	0.000000
1.045000	-18.330273
1.010000	-36.540585
0.951999	-25.451793
0.962696	-21.894412
1.070000	-29.177189
1.012687	-28.561894
1.090000	-28.561894
0.997437	-30.122894
0.992243	-29.931964
1.020836	-29.526091
1.026312	-29.653473
1.009243	-29.528441
0.948572	-30.351373

Table B.2 Comparison table when R/X ratio is increased

R/X ratio	First order	Method 1	Method 3	Method 4	Method 5a	Method 5b	Decelerated convergence method	Const. Positive shift algorithm	Increased const. Positve shift algorithm	Two step algorithm	Three step algorithm	Two step first order	Three step first order
1	4	4	4	4	4	3	4	4	4	3	2	4	2
2.5	5	4	5	5	5	5	5	4	4	-	3	-	-
3	-	4	-	6	6	6	-	4	4	-	-	-	-
4	-	6	-	7	12	13	-	6	6	-	-	-	-
4.5	-	-	-	-	-	-	-	-	-	-	-	-	-

Table B.3 Load flow solution for a 43 bus system

Voltage (p.u.)	Angle (deg.)
1.136000	0.000000
1.100000	-10.555487
1.100000	-15.082066
1.100000	-13.822404
1.100000	-10.708125
1.100000	-10.906184
1.100000	-14.271283
1.100000	-12.498899
1.100000	-12.601449
1.100000	-12.601449
1.100000	-12.601449
1.100000	-14.469342
1.100000	-13.701780
1.100000	-13.915105
1.100000	-10.847436
1.100000	-16.048927
1.100000	-12.601449
1.018945	-15.741844
1.100000	-15.237993
1.100000	-13.487309
1.100000	-12.803471
1.100000	-15.376954
1.100000	-12.536985
1.100000	-16.075484
1.100000	-13.099537
1.100000	-19.579128
1.100000	-14.779140
1.100000	-11.060861
1.100000	-12.576323
1.100000	-11.599102
1.000000	0.056597
1.000000	-6.683868
1.000000	-13.786386
1.100000	-18.262344
1.000000	-13.368033
1.100000	-18.349936
1.011615	-4.989112
1.100000	-18.262344
1.100000	-11.357462
1.100000	-11.817661
1.100000	-13.881186
1.100000	-16.729330
1.100000	-11.454702

Table B.4 Comparison table for 43 bus system by varying R/X value

R/X ratio → **Method ↓**	**1**	**2**	**6**	**7**
First order	4	4	7	-
Method 3	4	4	5	-
Method 4	7	7	8	-
Method 5b	5	5	5	5
Method 5c	7	-	-	-
Method 6	4	5	6	-
Decelerated Convergence	4	4	5	-
Two step algorithm	3	3	4	-
First order two step	3	3	6	-
Three step algorithm	2	2	3	-
First order three step	2	2	4	-
Const. Positive Shift algorithm	5	6	6	-
Increasing positive Shift algorithm	4	5	5	-

Table B.5 Load flow solution for a 31 bus system

Voltage (p.u.)	Angle (deg.)
1.000000	0.000000
0.964409	-2.764822
0.958553	-2.663226
0.946757	-3.004730
0.923483	-3.602011
0.913925	-3.857894
0.887420	-4.213421
0.872138	-4.439202
0.856870	-4.673033
0.847128	-4.840292
0.838056	-5.002509
0.810927	-5.510250
0.800000	-5.735566
0.800000	-6.194209
0.800000	-6.641472
0.854828	-4.729728
0.852855	-4.755128
0.851213	-4.776486
0.885276	-4.270584
0.880137	-4.408816
0.878189	-4.433364
0.887076	-4.222656
0.939247	-3.225490
0.935161	-3.346990
0.931425	-3.440946
0.921894	-3.683553
0.918112	-3.781600
0.915175	-3.817119
0.961885	-2.732260
0.955095	-2.810333
0.952779	-2.837267

Table B.6 Comparison table for 31 bus system by varying R/X value

R/X ratio → Method ↓	1	1.5
First order	5	-
Method 1	6	-
Method 3	5	-
Method 4	8	-
Method 5a	-	-
Method 5b	4	5
Method 6	5	-
Decelerated Convergence	5	-
Two step algorithm	4	-
First order Two step algorithm	4	-
Three step algorithm	3	-
First order Three step algorithm	3	-
Const. Positive Shift algorithm	5	-
First order Const Positive Shift algorithm	5	-
Increasing positive Shift algorithm	5	-
First order Increasing positive Shift algorithm	5	-

REFERENCES

1 S C Tripathy, G Durga Prasad, O P Malik and G S Hope ' Load flow solution for ill- conditioned power systems by a Newton's like method' IEEE Trans. Power Appar. & Syst. Vol 101 (1982) pp 3648-3657

2 G Durga Prasad, A K Jana and S C Tripathy ' Modifications to Newton-Raphson load flow for ill-conditioned power systems' IEEE Trans. Electrical power & Energy systems Vol 12, No. 3, July 1990 pp 192-196

3 Proposed by Dr. P K Kalra Professor, Department of Electrical Engineering, IIT Kanpur

4 I J Nagrath, D P Kothari (1997). ' Modern Power System Analysis'. Tata McGraw-Hill publishing company limited, New Delhi

5 Marija Ilic, Franscisco Galiana, Lester Fink (1998). ' Power system restructuring: Engineering and Econimics'. Kluwer academic publishers, London.

6 Grainger, J J and Civanlar, S (1985). ' Volt/var control on distribution systems with lateral branches using shunt capacitors and voltage regulators, part III: The mathematical results'. IEEE Transactions in power apparatus and systems, 104 (11).

7 Mohamed E El-Hawary (1983). 'Electric power systems design and analysis'. Reston publishing company, Inc., A Prentice-Hall Company, Reston, Virginia.

8 C L Wadhwa (1993). ' Electrical power systems'. Wiley eastern limited, New Delhi.